Algorithms for terrain and bathymetric sensor data

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Three algorithmic advances and a research topic in processing topographic and bathymetric sensor data:

- lossy terrain compression that maintains slope accuracy,
- bathymetric surface fitting to irregular tracklines,
- lossy compression of 5D environmental data, and
- terrain modeling to maintain hydrological validity.

Why? To attack several issues raised by the large amounts of data now available.

**Eventual goal:** A unified system.
Lossy terrain compression that maintains slope accuracy

- Accurate elevations $\nRightarrow$ accurate slopes
- Bad commercial slope representation.
- **Goal**: Compress and reconstruct terrain so that slope derived from reconstructed terrain is good.
Accurate elevations $\nRightarrow$ accurate slopes

- Ignoring errors, slope is simply $f'(x)$
- But $\limsup_{i \to \infty} |(f_i(x) - f(x))| \to 0$, gives no guarantees about $\limsup_{i \to \infty} |(f'_i(x) - f'(x))|$
- Consider two approximations to $y(x) = 0$

- Elevation got better but slope got worse.
Bad commercial slope representation

Commercial SW:
Bad commercial slope representation

Commercial SW:

Photo:
ODETLAP – Overdetermined Laplacian Method

Fundamental representation for this work

- Small set of posts $\Rightarrow$ complete matrix of posts
- Overdetermined linear system:
  - $z_{ij} = h_{ij}$ for known points,
  - $4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}$ for all nonborder points.
- Emphasize accuracy or smoothness by weighting the two types of equations differently.

- Original goal: fill contours to a grid w/o showing terraces; competing methods have these problems:
  - Information does not flow across contours $\Rightarrow$ slopes discontinuous
  - If rays are fired from the test point to the first known point, then method is not conformal etc.
ODETLAP Advantages

Handles

- missing–data holes.
- incomplete contours,
- complete contours,
- kidney–bean contours,
- isolated points,
- inconsistent data.
ODETLAP hard example

- input: contours with sharp corners
- output: smooth silhouette edges, inferred top
ODETLAP process

Input

400x400 matrix of elevations

ODETLAP point selection

contour lines

any user-supplied points, even inconsistent

Small point set ~1000

ODETLAP terrain reconstruction

Compressed distributed data

Reconstructed data

400x400 matrix of elevations
ODETLAP summary

Original Surface (320 KB)

Compressed Surface (4071 Bytes)

Average Absolute Error = 2.451
Maximum Absolute Error = 25.822
Slope definition, accuracy

- Zevenbergen-Thorne \( \left( (p_{i-1,j} - p_{i+1,j}) \times (p_{i,j-1} - p_{i,j+1}) \right)_z \)
- \( p_{ij} \) not used

Limits of slope accuracy

- 1m elevation resolution
- 30m post spacing
- slope precision: \( \arctan \left( \frac{1}{30} \right) \approx 3\% \approx 2^\circ \)

Info content

- Slope’s autocorrelation distance is smaller than elevation’s
- However, slope has less relative precision.
Level-II sample datasets

400 \times 400 elevation matrices, elevation range

Hill1 505m
Hill2 745m
Hill3 500m
Mtn1 1040m
Mtn2 953m
Mtn3 788m
Idea 1: Pin down the elevation at sets of close points

- When inserting a point into known set, also insert some adjacent points
- *Thesis*: that will force the slope to be accurate there.
- Not really.
- *Analogy*: Lagrangian interpolation.

Keep trying.
Idea 2: Extend ODETLAP

- Explicitly incorporate slope

New overdetermined linear system:

- unknowns: $z_{ij}$
- known:
  - some $h_{ij}$,
  - some $\Delta_x h_{ij} \triangleq h_{i-1,j} - h_{i+1,j}$,
  - some $\Delta_y h_{ij} \triangleq h_{i,j-1} - h_{i,j+1}$,
- for all nonborder points:
  $4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}$
- for known $h_{ij}$: $z_{ij} = h_{ij}$
- for known $\Delta_x h_{ij}$ and $\Delta_y h_{ij}$:
  $z_{i-1,j} - z_{i+1,j} = \Delta_x h_{ij}$
  $z_{i,j-1} - z_{i,j+1} = \Delta_y h_{ij}$
Preliminary results

Slope Accuracy vs. Compression

Elevation Accuracy vs. Compression
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Fitting bathymetric data is hard

- MBB data is very unevenly spaced (dense in a swath along the ship tracklines, but then nonexistent for a long distance sideways),
- depth accuracy is a few percent, and
- insufficient data to infer features that are probably there.

Current methods often

- have a specific distance wired into the formula,
- do not let information flow past data points, and so
- produce artifacts (e.g., abrupt slopes, acquisition footprint);
- show details that aren’t justified.
Sea floor bathymetry trackline fitting

*Problem:* Trackline data is very unevenly spaced, leading to very bad surface fitting.

Bathymetry Dataset  Kriging w. ArcGIS  Voronoi Polygons

Inverse Distance  2nd-order Spline Interp  Soln: ODETLAP, $R = 100 \rightarrow 10$
Relevant terrain property

- Terrain is unlikely to have created artifacts exactly where the multibeam bathymetry later scanned it.
- How to work this fact into the math?

ODETLAP extension:
- vary $R$ depending on distance to known points
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5D data compression

- Sensors, e.g., in World Ocean Atlas 2005, collecting multiple bands of environmental data –
  - temperature, salinity, oxygen concentration,
  - producing set of values over 5D grid \((x, y, z, t, b)\).
- Compress it!
- little prior art.

Principles:
- Assume one band’s large derivative at particular \((x, y, z, t)\) ⇒ likely for the other bands,
- Treat the data as one 5-D dataset, and
- Compress lossily since the data is imprecise.
Data compression technique

- extend ODETLAP to 3D, then 4D, 5D.
- **Major challenge**: Everything harder in higher dimensions.
- **To date**: compression ratios of 100:1 (mean error < 1.5%).

<table>
<thead>
<tr>
<th>Variable</th>
<th>3D-ODETLAP</th>
<th>3D-SPIHT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Err(%)</td>
<td>Max Err(%)</td>
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<tr>
<td>Salinity</td>
<td>0.0532</td>
<td>0.2174</td>
</tr>
<tr>
<td>Temperature</td>
<td>0.4993</td>
<td>2.0673</td>
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<tr>
<td>Dissolved $O_2$</td>
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<tr>
<td>Apparent $O_2$ util.</td>
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<td>4.0170</td>
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<tr>
<td>Percent $O_2$ satur.</td>
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<tr>
<td>Phosphate</td>
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<tr>
<td>Nitrate</td>
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<td>4.6946</td>
</tr>
<tr>
<td>Silicate</td>
<td>0.9996</td>
<td>5.1437</td>
</tr>
</tbody>
</table>

ODETLAP’s smaller compression error than SPIHT
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Maintaining hydrological validity

Given incomplete hydrography; fill in gaps.

- Presented on Wed
- Don’t work directly on partial hydrography.
- Compute deeper representation (terrain) from it.
- Derive hydrography.
- Result is guaranteed internally consistent.
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Disclaimer and goals

*This section is a report on work not yet done.*

**Goals:** Math that

- allows the representation of only legal terrain (= height of land above geoid),
- minimizes what needs to be stated explicitly, and
- enforces global consistencies.

**Why?** To put compression and other ops on a logical foundation.
Terrain properties

Messy, not theoretically nice.

- Often discontinuous ($C^{-1}$).
- Many sharp local maxima.
- But very few local minima.
- Lateral symmetry breaking — major river systems.

- Different formation processes in different regions.
- Features do not superimpose linearly; two canyons cannot cross and add their elevations.
- $C^\infty$ linear systems, e.g., Fourier series, are wrong.
- Structure that people can recognize even though hard to formalize; see Figure.
- Multiple related layers (elevation, slope, hydrology).
Current representations

• Array of elevation posts.
• Triangular splines, linear or higher.
• Fourier series.
• Wavelets

Theory vs practice:
• Slope is derivative of elevation, but
• that amplifies errors, and
• lossy compression has errors, so
• maybe we want to store it explicitly.
Also, shoreline is a level set, but...
Inconsistencies between layers

Elevation contours crossing shoreline
Slope is important

- mobility
- erosion
- aircraft
- visibility
- recognition
Path planning

Example of a common terrain operation. Cost depends on
- distance
- uphill climb
- being seen
- not a metric: $d(a, b) \neq d(b, a)$
- not a scalar field difference: $d(a, b) \neq -d(b, a)$

$$C = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \cdot \left(1 + \max \left(0, \frac{\Delta z}{\sqrt{\Delta x^2 + \Delta y^2}}\right)\right) \cdot (1 + 100v)$$
Smugglers and Border Guards
Math should match physics

- Fourier series appropriate for small vibrations, not terrain.
- Truncating a series produces really bad terrain.
- Anything, like Morse complexes, assuming continuity is irrelevant.
- Fractal terrain is not terrain.
- Wavelets: how to enforce long-range consistency?
- Topology, by itself, is too weak.
- Terrain is not linear, not a sum of multiples of basis function.
Examples of rich structure

- group theory
- trig
- constructive solid geometry
- line generalization from level sets
- hydrology from terrain
- polygon properties from local geometry and topology.

Line generalization:

Baarle-Nassau - Baarle-Hertog border
Terrain formation by scooping

- **Problem**: Determine the appropriate operators, somewhere inside the range from conceptually shallow (ignoring all the geology) to deep (simulating every molecule).

- **One solution**: **Scooping**. Carve terrain from a block using a scoop that starts at some point, and following some trajectory, digs ever deeper until falling off the edge of the earth.

- **Properties**: Creates natural river systems w cliffs w/o local minima.

- Every sequence of scoops forms a legal terrain.

- Progressive transmission is easy.
Terrain formation by features

- Represent terrain as a sequence of features — hills, rivers, etc..
- plus a combining rule.
- This matches how people describe terrain.
- Progressive transmission.
- The intelligence is in the combining rule.

How compact is this rep? How to evaluate it?
Implications of a better rep

- Put earlier empirical work on a proper foundation.
- Formal analysis and design of compression.
- Maximum likelihood interpolation, w/o artifacts.
- Treat more sophisticated metrics, like suitability for operations like path planning, or recognizability.
- Close the loop to pre-computer descriptive geometry.