The RPI GeoStar Project

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7 July 2011
History

- DARPA-sponsored project at RPI
- March 2005 — Jan 2009
- continuing my ideas of many years
- Faculty: WRF, Prof Barb Cutler, Prof Frank Luk
- PhD students completed: Dr Metin Inanc, Dr Dan Tracy
- PhD students in progress: Chris Stuetzle, Zhongyi Xie
- MS students: Jake Stookey, Jon Muckell
- Contractors: Joe Roubal (ESRI)
Tasks

- Terrain representation
  - Morphological terrain sculpting
  - Overdetermined Laplacian PDE (ODETLAP)
  - Triangulated irregular network (TIN)
  - Lossy compression of terrain and slope

- Terrain operators
  - Siting/intervisibility toolkit
  - Trajectory (aka path, motion) planning
  - Drainage analysis
Slope Accuracy on Compressed Terrain — Why consider slope?

Slope is important for

- mobility
- erosion
- aircraft
- visibility
- recognition
Bad commercial slope representation

Commercial SW:
Bad commercial slope representation

Commercial SW:

Photo:
Inconsistent layer representation
Accurate elevations $\nRightarrow$ accurate slopes

- Ignoring errors, slope is simply $f'(x)$
- But $\limsup_{i \to \infty} |(f_i(x) - f(x)| \to 0$, gives no guarantees about $\lim sup_{i \to \infty} |(f'_i(x) - f'(x)|$
- Consider two approximations to $y(x) = 0$

Elevation got better but slope got worse.
Fundamental representation for this work

- Small set of posts $\Rightarrow$ complete matrix of posts
- Overdetermined linear system:
  - $z_{ij} = h_{ij}$ for known points,
  - $4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}$ for all nonborder points.
  - Emphasize accuracy or smoothness by weighting the two types of equations differently.
- Original goal: fill contours to a grid w/o showing terraces; competing methods have these problems:
  - Information does not flow across contours $\Rightarrow$ slopes discontinuous
  - If rays are fired from the test point to the first known point, then method is not conformal etc.
ODETLAP Advantages

Handles
- missing–data holes.
- incomplete contours,
- complete contours,
- kidney–bean contours,
- isolated points,
- inconsistent data.
ODETLAP hard example

- **input**: contours with sharp corners
- **output**: smooth silhouette edges, inferred top
ODETLAP process

Input

400x400 matrix of elevations

ODETLAP point selection

400x400 matrix of elevations

Compressed distributed data

Reconstructed data

contour lines

Smaller point set ~1000

any user-supplied points, even inconsistent

ODETLAP terrain reconstruction
Goal is min size not fewest points

- What about not selecting points adaptively, but using a regular grid?
- More points, but all the \{(x, y)\} combined compress down to one byte
- However, small features get lost, and
- that fact is not captured by a simple RMSE metric.
• *Coding* \(\{(x, y, z)\}\) to minimize size is as important as selecting the points.
• Using more points is good, if they can be coded better.
• E.g., regular grid of points.
• If progressive transmission is not desired, then, for irregular points, use compressed bitmap (CCITT G4) for \(\{(x, y)\}\) and *bzip2* for \(z\).
Information theoretic limit for point coding

- Assume that 1000 of the $400^2$ bits are 1, the others 0.
- Assume no further structure.
- Info content: $\lg \left( \binom{160000}{1000} \right) = 8754$ bits.
- We are within 20% of that.
- That’s why we separate $\{(x, y)\}$ from $(z)$. 
Original Surface (320 KB)

Compressed Surface (4071 Bytes)

Average Absolute Error = 2.451
Maximum Absolute Error = 25.822
ODETLAP fills in missing data holes

The Original Dataset # = 20
Elevation Range [1803, 1898]

Filled with Overdet PDE
K = 0.1, R = 40, [1803, 1898]

Error on the fill
Range in Circle [1822, 1883]

3D Plot of the Filling
Slope definition, accuracy

- Zevenbergen-Thorne \( \left( (p_{i-1,j} - p_{i+1,j}) \times (p_{i,j-1} - p_{i,j+1}) \right)_z \)
- \( p_{ij} \) not used

Limits of slope accuracy

- 1m elevation resolution
- 30m post spacing
- slope precision: \( \arctan \left( \frac{1}{30} \right) \approx 3\% \approx 2^\circ \)

Info content

- Slope’s autocorrelation distance is smaller than elevation’s
- However, slope has less relative precision.
Level-II sample datasets

400 × 400 elevation matrices, *elevation range*

- **Hill1**: 505m
- **Hill2**: 745m
- **Hill3**: 500m
- **Mtn1**: 1040m
- **Mtn2**: 953m
- **Mtn3**: 788m
Idea 1: Pin down the elevation at sets of close points

- When inserting a point into known set, also insert some adjacent points
- *Thesis*: that will force the slope to be accurate there.
- Not really.
- *Analogy* Lagrangian interpolation.

Keep trying.
Idea 2: Extend ODETLAP

- Explicitly incorporate slope
- New overdetermined linear system:
  - unknowns: $z_{ij}$
  - known:
    - some $h_{ij}$,
    - some $\Delta_x h_{ij} \triangleq h_{i-1,j} - h_{i+1,j}$,
    - some $\Delta_y h_{ij} \triangleq h_{i,j-1} - h_{i,j+1}$,
  - for all nonborder points:
    \[4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}\]
  - for known $h_{ij}$: $z_{ij} = h_{ij}$
  - for known $\Delta_x h_{ij}$ and $\Delta_y h_{ij}$:
    \[z_{i-1,j} - z_{i+1,j} = \Delta_x h_{ij}\]
    \[z_{i,j-1} - z_{i,j+1} = \Delta_y h_{ij}\]
Mtn2 experiments

Slope error vs compressed file size

mtn2, ODETLAP with slope equations, lossy encoding of delta-z

Uncompressed binary file: 320KB
Incrementally Compressing LIDAR data

video: Dan8-10-08-1.m1v
Hi-res viewsheds

Can’t see through ridges
Viewshed uncertainty

A small change in the interpolation algorithm between adjacent posts changes the visibility of 1/2 the viewshed.

Hue indicates elevation

Visible

Possibly hidden

Hidden

Probably hidden

Observer
Lossy compression evaluation using observer siting

Original, hi-res, terrain ↓ Reduce horizontal or vertical resolution.

Compute multiple observer siting. → Best possible observer siting, given the data

Compute cumulative visibility. → Expensive cumulative visibility index

Transfer these observers to the hi-res terrain and compute their new cumulative visibility index.

Compute multiple observer siting. → Best observer set; computed on & evaluated on the lo-res terrain

Best observer set; computed on & evaluated on the lo-res terrain → Best estimate of the cheap cumulative visibility index

Compare.
Path planning

- Combines various projects we’ve worked on.
- *multiobserver siting* + *path planning* + *surface compression* with ODETLAP
- **unique feature of our path planning**: plans around complicated obstacles (viewsheds) while minimizing complex non-symmetric objective:
  - not a *metric*: $d(a, b) \neq d(b, a)$

\[
C = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \cdot \left( 1 + \max \left( 0, \frac{\Delta z}{\sqrt{\Delta x^2 + \Delta y^2}} \right) \right) \cdot (1+100v)
\]
Lossy compression evaluation with path planning

Smugglers’ Path Planning on 16x Compressed “Scooped” Terrain Representation

Original 3595x3595 W111N31 Terrain: 12,924,025 d.f., Elev Range=2071

Compressed (7x7 Scoop): 791,267 d.f. (16x reduction), Mean abs error=1.7 (0.1%).

Compressed: Shortest Smugglers Path Computed Avoiding All 324 Viewsheds of Optimally Sited Observers

Original: Joint Viewshed Computed for Same 324 Observers

Evaluation: Optimal Path from Compressed Terrain Tested on Original Terrain Viewsheds – 14 of 4767 Points (0.3%) Are Erroneously Visible
Many optimal paths

video: RPI-multipath.wmv
Path planning on Ottawa LIDAR data
Path planning video

video: RPI-path-planning
Future Ideas

• conflate global lo-res with local hi-res elevation
• play red-blue games with multi-observer siting and path planning, detect and block choke points
• new apps for siting: radio transmitters, micro cells, exit lights, surveillance camers
• urban multi-observer siting, in 3D
• conflation, compression and data fill-in of urban geometric data while preserving structure and the laws of formation
  • roads, rivers are continuous and usually don’t dead-end
  • size depends on catchment
• Long term goal: procedural terrain representation, where the math captures the structure.
• modify the real world to enhance the goal (visibility, motion, ...)

Commercialization

Two 2007.2 SBIRs clearly based on this research.

- **A07-123 Novel Representations of Elevation Data**
  Two phase I awards:
  - **W9132V-08-C-0012** to Andrews Space, Inc.
  - **W9132V-08-C-0013** to Numerica Corp.

- **A07-126 Optimal Intervisibility Site Selection**; cited me four times. Phase I award **W9132V-08-C-0005** to Toyon Research Corp.

(Unfortunately) I have no connection to any of those companies.
Followon projects

- Hosted the 18th Fall Workshop in Computational Geometry at RPI on Fri 10/31 and Sat 11/1/2008, with NSF support.
- Received an NSF *Cyber-enabled Discovery and Innovation (CDI) Fundamental Terrain Representations and Operations* award. (success rate: 2% (1 in 50)).
- Bathymetry: trackline fitting.
Sea floor bathymetry trackline surface fitting without visible artifacts using ODETLAP

*Problem:* Trackline data is very unevenly spaced, leading to very bad surface fitting.

Bathymetry Dataset  
Kriging Interpolation w. ArcGIS
More bad methods

Voronoi Polygons

Inverse Distance Weighting
Another bad, and ODETLAP

2nd-order Splinep Inter (w Matlab griddata)  ODETLAP, $R = 50$
Trackline data characteristics

- no wide-area data capture techniques such as IFSAR
- Multibeam Bathymetry produces many data points with 10\textit{m} resolution in a swath up to 10\textit{km} wide along a ship’s trackline.
- No data between tracklines.
- Test data: 10° × 10° region
- Want a 601 × 601 grid at 1′ cell resolution.
- 857 670 data points distributed among 23 630 grid cells.
- Average the average of 36 points per cell.
- 1/3 of cells had depth range > 100\textit{m}.
- Therefore features smaller than that may not exist.
- Eval that with a combo of contours and relief shading using Mathematica.
Extending ODETLAP to variable R

*Problem:* The true surface may have varying detail levels, but probably not correlated with the tracklines.

*Soln:* Vary $R$ (the relative weights of the two equation types) according to the distance to the closest known points.

*Result:* Surface shows more details but few artifacts.

ODETLAP, Variable $R$ from 10 to 100