Representation of Terrain Data Using a Mathematical “Drill” Operator

Christopher S. Stuetzle\textsuperscript{1} and W. Randolph Franklin\textsuperscript{2}

Rensselaer Polytechnic Institute

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\textsuperscript{1}PhD (RPI) 7/2012, now Assistant Prof (Merrimac College)

\textsuperscript{2}advisor
The Problem

Is there a terrain representation that
1. mimics physical manipulation of terrain by storing data procedurally?
2. facilitates prohibition of local minima?
3. allows encoding of complex features (caves, cliffs)?

Broader Impact:
More hydrographically valid datasets which can be compressed and manipulated for storage on mobile devices running GIS applications.
Terrain Properties

1. Few local minima (*basins*).
2. Large monotonic patterns (*rivers*).
3. Up and down different (*sharp peaks*).
4. Lateral symmetry breaking — major river systems.
5. Discontinuities (*cliffs*).
6. Land and water different.
7. Nonlinear (*overlaid features don’t add*).
8. Deleting features (*properly defined*) leaves terrain still valid, but less accurate.
Lossy Compression

1. The original data is very large, and
2. imprecise, so
3. lossy compression is indicated.

Problems:
1. rounds skylines
2. causes local minima
Popular Terrain Surface Representations

1. Height Fields (DEMs)
2. Triangulated Irregular Networks (TINs)
3. Fourier Series
4. Splines
5. Volumetric Representations:

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When pushed to limit, perhaps equally powerful.

Must separate abstract representation from coding (in data compression sense).

Tradeoff space and time.

TIN: if Delauney, then topology takes zero space (but using some bits makes it faster).

DEM: Leverage image processing techniques.
Then cut along an ever deeper path from the start to the edge of the world.
Nice Properties / Questions to Answer

1. Creates no interior minima.
2. Creates cliffs.
3. Creates hydrography.

1. How can we fit a drill to a terrain location (pixel) \( p \)?
2. How can we represent a terrain surface by a series of drill operations?
3. How can we regenerate a surface from a series of drill operations?
For several possible radii $r$:

1. Find $S$, terrain profile of size $r$
2. Fit function $F$ to $S$

More accurate $F$: less error but more space to represent.
Calculating Terrain Profile
Finding Best-Fit Polynomial for S

Elevation

0 Distance From p

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Representing Terrain $T$ with Drills

1. Extract the terrain’s channel network
2. Find best drill shape at each start, junction, and end point
3. Store channel segments and drills

Input

Extract Channel Network

Network

Fit Drills

Output: Drills and Segments

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Counterintuitive:

Even if original terrain has no basins, and the DEM would perfectly sample\textsuperscript{3} the elevation at each post.

The DEM has many basins.

Narrow rivers run between adjacent posts.

Fake basins will quickly trap all rivers, unless removed.

Eddie Lau will talk in Session 17 on Tues on, *Improving river network completion under absence of height samples using geometry-based induced terrain approaches.*

\textsuperscript{3}gr: subjunctive mood, indicating an impossible event
Regenerating the Terrain

\[ T_0 (X, Y) = \infty \]
\[ T_i = \min (T_{i-1}, D_i) \]

where \( T_i \) is the \( i^{th} \) step of the terrain regeneration, and \( D_i \) is the \( i^{th} \) drill matrix, an \( X \times Y \) matrix of elevation values resulting from evaluating \( F_i \) at each pixel’s distance from \( p \).
How to measure error

Root Mean Squares Error: *(obvious)*

\[
RMSE (T_0, T_1) = \sqrt{\frac{\sum_x \sum_y \left( T_0 (x, y) - T_1 (x, y) \right)^2}{N^2}}
\]

Ridge-River Error: *(good)*

\[
\begin{align*}
EnergyDown &= \sum_x \sum_y \max \left( 0, T_0 (x, y) - T_0 (r (x, y)) \right) \\
EnergyUp &= \sum_x \sum_y \max \left( 0, T_0 (r (x, y)) - T_0 (x, y) \right) \\
RRE (T_0, T_1) &= \frac{EnergyUp}{EnergyDown}
\end{align*}
\]

where \( r (x, y) \) is the pixel receiving flow from \((x, y)\).
Height Fields Used in this Work

MTN1

MTN2

MTN3

HILL1

HILL2

HILL3
## Accuracy Tests

<table>
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<tr>
<th></th>
<th>RMSE</th>
<th>SSE</th>
<th>RRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>HILL1</td>
<td>6.62 m</td>
<td>9.38°</td>
<td>0.012</td>
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<tr>
<td>HILL2</td>
<td>10.69 m</td>
<td>16.38°</td>
<td>0.008</td>
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<tr>
<td>HILL3</td>
<td>6.80 m</td>
<td>7.20°</td>
<td>0.015</td>
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<tr>
<td>MTN1</td>
<td>7.05 m</td>
<td>14.65°</td>
<td>0.003</td>
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<td>MTN2</td>
<td>6.80 m</td>
<td>14.31°</td>
<td>0.003</td>
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<td>10.41 m</td>
<td>15.51°</td>
<td>0.005</td>
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</table>

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**Terrain Compression Using the Drill Operator**

1. Compress each channel network segment
   - 1. Freeman Chain Codes
   - 2. Line Generalization
2. Encode each drill
3. Compress the data (binary encoding)
4. (Optional) Compress with archiving algorithm, such as 7Zip.
Terrain Compression Using the Drill Operator

<table>
<thead>
<tr>
<th></th>
<th>RMSE</th>
<th>SSE</th>
<th>RRE</th>
</tr>
</thead>
<tbody>
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<td>0.023</td>
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<td>MTN3</td>
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<tr>
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<th>Binary</th>
<th>ASCII</th>
<th>Drill</th>
<th>7Zip</th>
<th>PNG</th>
<th>JPEG</th>
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<td>22.0</td>
<td>116.0</td>
<td>158.6</td>
<td>39.9</td>
</tr>
</tbody>
</table>
Conclusion

The drill operator is already (although a work in progress) a viable representation of terrain data that

1. guarantees hydrographically valid terrain with no pits,
2. and comparable in size to JPEG with the similar error.
3. can represent complex terrain features (anything modeled with a mathematical curve).
4. stores elevations procedurally.
5. can be used to compress features of the terrain where other schemes oversmooth the surface.
Future Work

1. Additional drill shapes, and use 3D terrain data to model caves and cliffs.
2. Automatic detection of ideal drill parameters.
3. Temporal and spatial optimization.
4. Application to siting, progressive transmission, etc.
5. (Lofty) Develop in conjunction data collection methods that take advantage of drill flexibility.
6. Other terrain formation operators (*faults, volcanos?*).
7. When only *legal* terrain can be generated by a random system, with probability distribution, then siting and terrain operations can begin to be analyzed formally.
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We would like to thank the reviewers for their helpful comments. Our test data and code are freely available for nonprofit research and education.
Thank you! Questions?
Complex Terrain Formations

Surface representations cannot model complex terrain formations:

1. Cliff faces
2. Caves
3. Overhangs

Volumetric representations have a large memory footprint and are dependent on grid resolution.
Post-Process $T_{new}$

ODETLAP (OverDETermined LAPlacian) applied to $T_{new}$:

$$T(x, y) = \frac{T(x+1, y) + T(x-1, y) + T(x, y+1) + T(x, y-1)}{4}$$

$T(x, y) = T_s(x, y)$
Calculating $S$

\[
S = \text{zeros} \left( r + 1 \right)
\]

\textbf{for} \ p_j \in R \ \textbf{do}

\textbf{if} \ S \left[ \text{floor} \left( d \left( p, p_j \right) \right) \right] < T \left( p_j \right) \ \textbf{then}

\quad S \left[ \text{floor} \left( d \left( p, p_j \right) \right) \right] = T \left( p_j \right)

\textbf{end if}

\textbf{if} \ S \left[ \text{floor} \left( d \left( p, p_j \right) + 0.999 \right) \right] < T \left( p_j \right) \ \textbf{then}

\quad S \left[ \text{floor} \left( d \left( p, p_j \right) + 0.999 \right) \right] = T \left( p_j \right)

\textbf{end if}

\textbf{end for}
Determining $D_i$

\[
\text{for } j \in |N| \text{ do}
\]

\[
\text{for } k \in |N^j| \text{ do}
\]

\[
t \leftarrow \frac{k - 1}{|N^j| - 2}
\]

\[
D^j_k \leftarrow (t - 1) \ast D^j_0 + t \ast D^j_{|N^j|}
\]

\[
\text{end for}
\]

\[
\text{end for}
\]

For the $k^{th}$ pixel of the $j^{th}$ segment of our network, $N^j$, interpolate the drill matrix between the source drill ($D^j_0$) and sink drill ($D^j_{|N^j|}$).
Post-Process $T_{new}$

PostProcessTerrain(terrain $T$, drillSet $D$)

$oldT \leftarrow \text{zeros}()$

while $oldT \neq T$ do

$oldT \leftarrow T$

for $d \in D$ do

$T \leftarrow \text{applyDrill} (d, T)$

end for

$\text{ODETLAP} (T)$

end while

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