$P(A) = 0.1$

$P(B \mid A) = 0.9$ 

$P(A \mid B) = P(B \mid A) P(A) \div 0.1 \times 0.09$ 

$P(A' \cap B) = P(B \mid A') P(A') \div 0.99 \times 0.099$
\[ P(B) = P(B|A) P(A) + P(B|A') P(A') \]

\[ 2 = 0.999 + 0.009 \]

\[ 2.098 \]

\[ P(A \cap B) = P(A|B) P(B) \]

\[ P(B) = P(A \cap B) + P(A' \cap B) \]

\[ P(A|B) = \frac{P(A \cap B)}{P(B)} \]

\[ 2.098 \]

\[ 1.1 \]
P(A_1) = .2, \quad P(B|A_1) = .05
\quad\quad P(A_2) = .3, \quad P(B|A_2) = .03
\quad\quad P(A_3) = .5, \quad P(B|A_3) = .1

What is \( P(A_3 | B) \)?

\[
P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)
\]
\[
P(B) = .05 \times .2 + .03 \times .3 + .01 \times .5 = .01 + .009 + .005 = .024
\]

P(B and A_3) = B(B|A_3) P(A_3) = .005

\[
P(A_3 | B) = \frac{P(B \text{ and } A_3)}{P(B)} = \frac{.005}{.024} \approx .2 \text{ approx}
\]
independence of 3 events
def: indiff \( P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C) \)

book ex 2.33, 2.32 whatever
\( A \text{ and } B \text{ and } C \) Prob = 0

\[ P(A) \cdot P(B) \cdot P(C) = 1/8 \neq 0 \]

A, B and C not indep.

\[ P(A \text{ and } B) = P(A \mid B) \cdot P(B) = P(A) \cdot P(B) \text{ if indep} \]

so indep -> \( P(A \mid B) = P(A) \)

triple indep: does this imply pairwise indep?
event: is this pixel black?  \( P(B) = 0.01 \).
\( P(B') = 0.99 \)

look at 2 pixels:  
\[ P(\text{exactly 0 black}) = \binom{2}{0} \cdot 0.01^0 \cdot 0.99^2 = 0.98 \]
\[ P(\text{exactly 1 black}) = \binom{2}{1} \cdot 0.01 \cdot 0.99 = 0.02 \]
\[ P(\text{exactly 2 black}) = \binom{2}{2} \cdot 0.01^2 \cdot 0.99^0 = 0.0001 \]

\( 0.98 + 0.02 + 0.0001 = 1 \) (2 signif digits)

Geometric dist: repeat bernoulli trial until success.

Take a fair coin.
\( P(\text{H on 1st toss}) = p = 0.5 \)
\( P(\text{1st happens on 2nd toss}) = (1-p) \cdot p \)
\( P(\text{1st head happens on N-th toss}) = (1-p)^{(N-1)} \cdot p \)
\( P(\text{it will take at least N tosses}) = \sum_{i=N}^{\infty} \)

That has a simple formula.