3.11
a) Pmf x: P(x=0) = (1/2)^4 = 1/16.
P(x=1) = P(1,1)+P(1,0)+P(0,1) = (1/2)^2 = 1/4.
P(x=2) = P(2,0)+P(2,1)+P(2,2)+P(0,2)+P(1,2) = 1/16+1/16+1/16+1/16+1/16 = 7/16.
Pmf y: P(y=0) = P(TT) = 1/4
P(y=1) = P(TH)+P(HT) = 1/4 * 2 = 1/2
P(y=2) = P(HH) = 1/4

Pmf are different because the sample space is different. There are 9 different conditions for both Michael and Carlos, which are (2; 2); (2; 1); (2; 0); (1; 2); (1; 1); (1; 0); (0; 2); (0; 1); (0; 0) with corresponding conditions. Y only considers the outcome of one person but X considers both. Therefore, the different random variables choose different components of events and their probabilities.

b) Now for carlos P(H=0) = 14 * 14 = 1/16.
P(H=1) = 1/4 * 3/4 * 2 = 3/8
P(H=2) = 3/4 * 3/4 = 9/16

Now pmf x:
P(x=0) = P(0,0) = 1/16 * 14 = 1/64 //here the first value is the number of head carlos has.

Second is for Michael
P(x=1) = P(1,0)+P(1,1)+P(0,1) = 3/8 * 1/4 + 3/8 * 1/2 + 1/16 * 1/2 = (3 + 6 + 1)/32 = 5/16
P(x=2) = P(2,2)+P(2,0)+P(2,1)+P(0,2)+P(1,2)
= 9/16 * 1/4 + 9/16 * 1/4 + 9/16 * 1/2 + 1/16 * 1/4 + 3/8 * 1/4 = 43/64

3.53
N geometric, n = 1, 2, ...

\[ P[N = k | N \leq m] = \frac{P[N = k, N \leq m]}{P[N \leq m]} = \frac{P[N = k]}{P[N \leq m]} \]

\[ = \frac{1 - p}{1 - (1-p)^k} - \frac{1 - p}{1 - (1-p)^m} \leq k \leq m \]

\[ P[N \text{model}] = \sum_{j=0}^{\infty} p(j-p)^{2j+1} = (p(1-p) \sum_{j=0}^{\infty} (1-p)^j)^2 = \frac{p(1-p)}{1 - (1-p)^2} \]
\[ P[\text{pass the test}] = (1 - p) + p(1 - \alpha) \]
\[ P[\text{fail the test}] = p\alpha \]
\[ P[k \text{ items}] = [(1 - p) + p(1 - \alpha)]^{k-1}(p\alpha)^1 \]

3.90 We want to find \( n \) so that the 2nd arrival \( \overline{x} \) after more than 2 minutes 20% of the time:

\[ P[N(2) \leq n] = 0.90 = \sum_{k=0}^{\infty} \frac{2^k}{k!} e^{-2} \]

By trial and error we find \( n = 5 \).