PROB C3 4/16/20 P1

$X$: RV - $\mu$ not known

$X_i$: i.i.d.

Sample mean: $M_n = \frac{\sum X_i}{N}$

$M_n$ estimates $\mu$

$M_n = RV.$

$M_n$ has mean, var.

What are they?

$M_n = \frac{\sum X_i}{N}$

$X_i: \mu, \sigma$

$\sum X_i: N_m, \frac{\sigma}{\sqrt{N}}$

$M_n: \mu, \frac{\sigma}{\sqrt{N}}$
What's prob that $M_n$ is within $\varepsilon$ of $\mu$?

Try Chebyshev Inequality

$$P \left( \left| X - \mu \right| > 2 \sigma \right) \leq \frac{\sigma^2}{\varepsilon^2}$$

$$P \left( \left| M_n - \mu \right| > 2 \right) \leq \frac{\sigma^2}{\varepsilon^2}$$

$$= \frac{\sigma^2}{N \varepsilon^2}$$

Ex 7.9

$X_i = N + N_i$

We want $N \in P \left( \left| M_n - \mu \right| < \varepsilon \right)$

$N_i : \sigma = \mu \nu$

$M_n : \sigma = \sqrt{\frac{1}{N} \mu \nu}$

$P \left( \left| M_n - \mu \right| > 1 \right) \leq \frac{\varepsilon}{N}$
\[ 1 - \sigma^2 = 1 - 0.99 = 0.01 \]

\[ N = \frac{1}{2} \]

\[ N = 100 \]

\[ \text{Ex 7.10: How many times to toss an unfair coin so we probably know its bias pretty closely?} \]

"Pretty closely" = within 10%

"Prob" = 95% of time

\[ N \text{ tosses} \]

\[ RV = \# \text{Heads seen} \]

\[ \mu = Np \quad p \text{ unknown prob} \]

\[ \text{VAR: } Np(1-p) = \frac{N}{4} \]
EX 7.11

\[ X_1 = 1^{st} \text{ Customer Order Cost} \]

\[ E[X_1] = 8 \quad \text{STD} \{X_1\} = 2 \]

\[ S_{100} = \frac{X_1}{3} \]

\[ M\{S\} = 800 \]

\[ \text{STD}\{S\} = \sqrt{5} = 2.23 \]

Want \( P\{S > 840\} \)

\( S: \text{Gaussian} \quad \mu = 800 \quad \sigma = 223 \)

\[ P\{S > 840\} = Z(2) = 0.02 \]

\[ P\{780 \leq S \leq 820\} = \frac{2}{3} \]

\( \mu = \frac{\mu + 2\sigma}{3} \quad \mu + 2\sigma \)
EX 2.12

N unknown

\[ S_n : \mu = 8N \quad \sigma = 2N \]

1000 :

CONVERT 1000 TO Z SCORE:

\[ P(z = \frac{1000-8N}{2N}) = 0.9 \]

\[ z = 1.3 \]

\[ 2 \approx 1.3 \]

\[ 1.3 = \frac{1000-8N}{2N} \]

\[ N = 129 \]

FINAL LAST CLASS 4/27