Sampling with replacement

\{w/d\} ordering.

\(Q = \text{quant}}\)

\(D_r = \text{DIM}_r\)

\(N = 2\) coins in urn
Draw \(k = 2\) coins.

\(w/o\) repl. \(QP D_2\)
\(w/\) ord. \(DP\)

\(w/o\) repl. \(EQ\) \(DS\)
\(w/\) ord. \(EQ\) \(DS\)

\(w/\) repl \(w/\) order \(4 N_k\)
\(w/\) repl \(w/o\) order: \(3,0,0,0,0,0,0,0\)
8. 3 balls from 5 with repl.

1st: 5 choices
2nd: 5
3rd: 5

125 choices.

# cases where diff: $5 \times 4 \times 3 = 60$

Prob all diff: $\frac{60}{125}$

Permutations: # ways to order n objects

$3! = 6$: ABC, ACB, BAC, BCA, CBA, CBA
For each crash, which month? How many ways?

12¹ (if crashes all in different months)

12² (if don't care)

\[ P(\text{12 crashes all in diff. months}) = \frac{12!}{12^{12}} \]
MULTINOMIAL EXAMPLE

BINOMIAL

N = 4 COINS

WAYS TO DIVIDE INTO 2 SETS OF 2.

4 WAYS TO PICK 1 SET

3

2

= 6

2 WAYS EACH SET PICKED

\[ \binom{N(N-1)-(N-K+1)}{K} = \frac{N!}{(N-K)!K!} = \binom{N}{K} \]

DIVIDE INTO SETS OF 3 AND OR 1

\[ \binom{4}{3} = \frac{4!}{3!1!} = 4 \]
MULTI

DIVIDE 4 COINS INTO

3 SETS: 5/12 = 2, 1, 1

(4) = 4, 1

(2 1 1) = 2, 1, 1

4 coins, G, D, N, T:

Q, D, P, N

NP, 4, D

TOTAL 12
9 BALLS
WANT TO DIVIDE INTO 3 SETS
5, 2, 2 = 4, 3, 2
# WAYS?

\[
\frac{9 \cdot 8 \cdot 7 \cdot 6}{24} = \frac{9!}{4!5!} = \text{WAYS TO PICK 1ST SET}
\]

5 BALLS LEFT
5, 4, 3 = 5, 4, 3

\[
\frac{5!}{3!2!} = \text{WAYS TO PICK 2ND SET}
\]

1 WAY TO PICK 3rd

\[
\frac{9\cdot 8\cdot 7}{4!5!3!2!} = \frac{9\cdot 8\cdot 7}{4!5!3!2!} = \left(\frac{9}{4!5!3!2!}\right)
\]

\[
\frac{9\cdot 8\cdot 7}{5!4!3!2!} = \frac{9\cdot 8\cdot 7}{5!4!3!2!} = \left(\frac{9}{5!4!3!2!}\right)
\]

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\frac{9\cdot 8\cdot 7}{4!5!3!2!} = \frac{9\cdot 8\cdot 7}{4!5!3!2!} = \left(\frac{9}{4!5!3!2!}\right)
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\]

\[
\frac{9\cdot 8\cdot 7}{4!5!3!2!} = \frac{9\cdot 8\cdot 7}{4!5!3!2!} = \left(\frac{9}{4!5!3!2!}\right)
\]
BINOMIAL PROB

FAIR COIN \[ P(\text{HEAD}) = \frac{1}{2} \]

TOSSED \( N = 2 \) \[ P(TT) = \frac{1}{4} \]
\[ P(HT, TH) = \frac{1}{2} \]
\[ P(HT) = \frac{1}{4} \]

WAYS TO GET EXACTLY \( K \) HEADS = \( \binom{N}{K} \)

TOTAL # OUTCOMES = \( 2^N \)

\[ P(\text{EXACTLY } K \text{ HEADS}) = \frac{\binom{N}{K}}{2^N} \]

\( N = 4 \) \[ \binom{4}{K} = 1, 4, 6, 4, 1 \]

6 WAYS TO GET 2 HEADS

\[ P(2H) = \frac{6}{16} = .38 \]
\[ P(0\text{H}) = \frac{1}{16} \]
UNFAIR COIN

\[ P = \frac{2}{3} \text{ of head.} \]

Toss N times
See K Heads

\[ P(N=2, K=0) = \frac{1}{3} \cdot \frac{1}{3} = (1 - P)^N \]

\[ P(N, K) = \]

PC K HEADS IN A ROW, THEN NK TAILS (IN N+ROW)

\[ = P^k (1 - P)^{N-k} \]

\[ P(K \text{ HEADS, ANY ORDER}) = \binom{N}{K} P^K (1-P)^{N-K} \]

\[ \text{Prob of Tail in 1 Toss} \]

\[ \begin{array}{c}
N=4 \\
k=2 \\
p=\frac{2}{3}
\end{array} \]

\[ \left( \frac{2}{3} \right)^2 \cdot \left( \frac{1}{3} \right)^2 = \frac{4}{81} \approx 0.05 \\
\]

\[ \text{Prob of Tail in 1 Toss} \]

\[ N=4, k=4 \left( \frac{2}{3} \right)^4 \approx 0.2 \]
NON CAN EXAMPLE:
BUILD WIDGETS, P=0.1 BAD
P(2 OF 10 WIDS) BAD
\( \binom{10}{2} (0.1)^2 (0.99)^8 \)

MULTINOMIAL
PROB OF SELECTING A COLOR

PICK A COLOR RANDOMLY

PICK 2 COLORS

G = 0.3
B = 0.1
O = 0.6
Y = 0.3
P = 0.2

Pick 10 colors.

\[ P[\text{ALL BLUE}] = (1)^{10} \]

\[ P[2 \text{BLUE}, 2 \text{RED}, 2 \text{YELLOW}, 2 \text{PINK}, 2 \text{OY}]
\]

\[ \left( \frac{10}{2 \times 2 \times 2 \times 2 \times 2} \right) \times 1^2 \times 3^2 \times 3^2 \times 2^2 \times 1^2 \]

\[ \frac{10!}{2! \times 2! \times 2! \times 2! \times 2!} \]

Playing Funny Card Game.

\[ N = 52 \text{ cards} \]

\[ P(4) = \frac{15}{52} \]

\[ P(2 \text{H}, 1 \text{C}, 2 \text{D}) \]

\[ \left( \frac{\binom{5}{2} \times (\frac{15}{52})^2 \times \binom{20}{2} \times \frac{2}{52} \times \frac{7}{52} \right)^2 \]
\[
\binom{5}{2} = \frac{5!}{2!1!2!}
\]

Ways to divide a set of 5 things into a set of 2 and 1 and 1

For set of 2: 5 ways to pick 1st + 4...

\[ \frac{1}{2} \text{ because each set can be ordered 2 ways.} \]
5 things: A, B, C, D, E

10 ways to pick a set of
10 sets of 2 from that

A B C D E
B C D E
C D E
D E
E

1 + 3 + 2 + 1 < 10

5 ways to pick 1st

\( \frac{5}{20} \times \frac{4}{19} \times \frac{2}{18} \times \frac{1}{17} \times \frac{3}{16} \times 2 = 1 \)

\( \frac{1}{2} \) ways to pick any set
Pick 2 sets from 3

Each set was picked twice.

So divide by 2.

3 sets of 4 you can pick from 3

\[
\frac{N!}{(N-k)!k!} = \binom{N}{k}
\]

N=3, k=2
Toss 6-sided die

1. Fair \( P(1) = P(2) = \ldots = \frac{1}{6} \)

Toss 10 times

\[ P(1,1,2,2,3,3,4,4,6,6) \]

\[
\binom{10}{1,2,3,4,6}
\]

\[
\left( \frac{1}{6} \right)^{10}
\]

\[ \text{3 cannot up 3 times} \]

\[ \text{2 unfair} \quad P(1) = 50\% = 0.5 \]

\[ P(2) = P(3) = \ldots = P(6) = 0.1 \]

\[
\binom{10}{1,2,3,4,6}
\]