PROB CLASS 10
7/2/14/22

From 2020 Exam

5 Smokers Numbers Made Up

Cancer

C = Cancer

\[ P(c=15) = .1 \]

\[ P(s|c) = .9 \]

\[ P(c = 25) = P(c=15)P(s) \]

\[ = P(s|c)P(c) \]

\[ P(c) = \frac{P(c=15)P(s)}{P(s|c)} = \frac{.1 \times .9}{.8} \]

0.22

11
\[ p(c(s')) = ? \]
\[ p(c(s')) = p(c \land s') = p(s'|c) \cdot p(c) \]
\[ p(c(s')) = p(s'|c) \cdot p(c) \]
\[ p(c(s')) = -1 \cdot 0.022 = -0.022 \]

**Geometric Prob**

Taking hard class at MIT

\[ P = \text{prob pass} = \frac{1}{2} \]

Retake it. Pass is independent of previous attempt.
Prob: Given that \( k \geq p \geq 2 \) \( p = \frac{1}{2} \), \( -p = \frac{1}{2} \), \( p(x) = x^{-\frac{1}{2}} \)

\[ f(k) = \frac{1}{p} = 2 \]

\[ \sum_{k=1}^{\infty} p q^{k-1} = p \sum_{k=1}^{\infty} k q^{k-1} \]

Use derivation:

\[ \sum_{k=1}^{\infty} p q^{k-1} = \frac{d}{dq} \sum_{k=1}^{\infty} q^k = \frac{q}{1-q} \frac{1}{1-q} \]

\[ \sum_{k=1}^{\infty} k q^{k-1} = \frac{d}{dq} \frac{q}{1-q} = \frac{1}{(1-q)^2} \]
3. 20-SIDE DICE TOSS. SEE K < RANDOM VARIABLE OUTCOME.

EVENTS $A = \{1 \leq k \leq 10\}$

$B = \{k \in \{3, 5, \ldots, 18\}\}$

$C = \{3, 4, 6, 8, 10, 11, 13, 15, 17, 19\}$

ARE $A, B$ INDEPENDENT?

$p(A) p(B) = p(A \cap B) \frac{1}{4}$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{5}{4}$

$A \cap B = \{3, 5, 7, 9\}$

$A, C$ are

$A \cap C = \{9, 4, 6, 10\}$

$p = \frac{5}{4}$
Table 3.1 on page 115-116 is important. Discrete

Table 4.1 page 164 continuous

Cauchy R.V.
\[ f(x) = \frac{1}{\pi} \frac{1}{1 + x^2} \]
\[ \alpha = x = \infty \]

How it can happen

Spin a pointer centered at \((0, 1)\)

Look at where it hits the axis.

That's R.V.

\[ \int_{-\infty}^{\infty} f(x) \, dx = 1 \]
Tails are thicker than for Gaussian:

\[ E[X] = \int_{-\infty}^{\infty} x f(x) \, dx - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{x}{(1+x^2)^{3/2}} \, dx \]

Diverges:

\[ E[X] \text{ does not exist.} \]

If you sample outcomes:

\[ x_1, x_2, \ldots, x_n \]

Sample mean \( \bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} \)

Converges to \( E[X] \).
MEANING OF $E[f]$. 

For most probability distributions, you can take a sample of $N$ r.v. and compute sample mean $E[\bar{x}]$.

As $N$ grows, sample mean tends to settle down.

For Cauchy, mean of large sample does not settle down.

$E[f]$ does not exist.

Neither $V(S)$
In real world maybe some economics models also are invalid.

See "Long term capital management"

Back to 2020 exam.

\[ A = \{0 \} \]
\[ P(A) = \frac{1}{2} = P(B) = P(C) \]
\[ B = \{ 0, 5 \} \]

\[ AnB \cap C = \emptyset \leq 5 \]
\[ P(A \cap B \cap C) = 0 \]
\[ \neq P(A)P(B)P(C) \]

Knowing A tells you nothing about C.

Knowing that both A, B happened tells that C is impossible.

\[ P(C|A) = \frac{1}{2} \]

Bayes
Display with 2000² pixels. Manufacturer defined good display as ≤10 bad pixels.

\[ P(\text{a given pixel being bad}) = 10^{-6} \]

Pixels independent

**APPROPRIATE PROBAB DISTN?**

**POISSON**

**POISSON** \[ \text{Prob \ of \ } k \text{ bad} \quad N = 4000 \text{, } 000 \]

\[ p = 10^{-6} \quad \binom{N}{k} p^k (1-p)^{N-k} \]

**POISSON** \[ \lambda = 4 \]

\[ \mu = 4 \]

\[ \text{var} (k) = 4 \]

\[ \sigma = 2 \]

2/3 of times, we have 2 to 6 bad pixels.

\[ p(k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad p(0) = e^{-\lambda} \]
What fraction of displays are acceptable?

\[ \sum_{k=0}^{10} p(k) = e^{-x} \sum_{k=0}^{4} \frac{4^k}{k!} \]

Back to Chap 4

\[ \text{Gaussian (Normal)} \]

\[ f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad -\infty < x < \infty \]

CDF = \int_{-\infty}^{x} f(t) dt = \Phi(x)

\[ \Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \]

\[ \Phi(x) = (\Phi(x)) \quad \text{RIGHT TAIL} \]
TRANSFORM FOR OTHER \( \mu, \sigma \).

\[ \mu = 5, \sigma = 1 \]

WANT \( P[X = 500] \)

\( Q(0) = \frac{1}{2} \)

\( P[X = 400] = Q(-1) = (-\Phi(1)) = 5 \)
\[ f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

Apply noisy channel

\[ \text{INPUT} \rightarrow \text{OUTPUT} \mid P(0) = p \]

\[ X \rightarrow Y = 1 \cdot \text{input} \]

Add noise \( N \) (it’s Gaussian \( N(0,1) \))

Received signal

\[ Y = X + N \]
GUESS WHAT WAS TRANSMITTED.

\[ \mathbb{P}(Y) = \mathbb{P}(Y|X=0) \mathbb{P}(X=0) + \mathbb{P}(Y|X=1) \mathbb{P}(X=1) \]

\[ \mathbb{P}(Y|X=0) = \mathcal{N}(0,1) \]

\[ \mathbb{P}(Y|X=1) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(Y-\mu)^2}{2\sigma^2}} \]

\[ Y = X + N = 1 + N \]

\[ \mathcal{N}(\mu, \sigma^2) \]

\[ \mathcal{N}(\mu_0, \sigma^2) \]

\[ N \text{ is Gaussian noise R.V. added to transmit. Also } N(\mu_0) \text{ is Gaussian R.V.} \]