Data Structures for Parallel Spatial Algorithms on Large Datasets

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Abstract

- Efficient parallel data structures are different:
  - *Suboptimal: trees, recursion, pointers, sweep lines, global topologies.*
- 2D GIS and 3D CAD share a lot—learn from each other.
  - *In additive manufacturing (3D printing), easier to build than to analyze.*
- New way of looking at geometry is useful.
- This talk:
  - local geometric data structures for map–reduce.
  - local parallel computing.
- Big example: overlay two triangulated polyhedra, total 5.7M triangles in 5.5 real seconds on 16 core Xeon workstation.
  - *That used rational numbers to prevent roundoff, and Simulation of Simplicity to handle geometric degeneracies, or it would have been even faster.*

Our prior parallel geometry implementations. . .

on multicore Intel Xeon, with OpenMP

- Volume of union of 100M identical cubes (2003)
- 2D planar graph overlay (BIGSPATIAL 2015)
- 3D point location (Berlin Geometry Summit 2016)
- Triangulated polyhedra overlay (IMR 2018)

on Nvidia GPUs, with Thrust

- Find all pairs of 3D points closer than given δ (BIGSPATIAL 2017)
- Preprocess points in 2D to 6D for nearest point query (CCCG 2016)

Background

- Philosophically a Computer Scientist.
- PhD officially in Applied Math.
- Working in Electrical, Computer, and Systems Engineering Dept.
- Students are from Computer Science.
- Teaching Engineering Parallel Computing.
- Collaborating with Geographers for a long time.
- Enjoy applying computer science and engineering to GIS.

Historical analogy

- Roebling, builder of Brooklyn Bridge, graduated from RPI.
- 15 year project.
- after spending money for 2 years, there was no visible progress.
- Roebling was building the foundations.
- None of his bridges ever fell down.
- In contrast: In last few decades, three interstate highway bridges have collapsed from design errors compounding maintenance lack.

Spend some time on the foundations.

Massive shared memory

- An underappreciated resource.
- External memory often not needed.
- Paging virtual memory is obsolete.
- Inexpensive servers have 1TB of memory.
- Even for Nvidia GPUs:
  - up to 48GB,
  - several can be ganged together with hi-speed bus.
- Many problems don't require the overhead of—
  - MPI,
  - supercomputers,
  - distributed cloud computing.

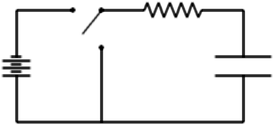
Parallel computing

- Multicore Intel Xeon underappreciated.
  - *Dual 20 core: 80 hyperthreads.*
- One Xeon core is $20\times$ more powerful than one CUDA core.
- Nvidia GPUs: up to 5000 cores, 48GB memory.
- Lower clock speed 750MHz vs 3.4GHz
- Hierarchy of memory: small/fast $\longleftrightarrow$ big/slow
- Communication cost $\gg$ computation cost
- Preferred: blocks of threads execute SIMT.
- Top 500 OS: never always some variant of

Why parallel HW?

- More processing $\rightarrow$ faster clock speed $\rightarrow$ more electrical power. *Each bit flip (dis)charges a capacitor through a resistance.*
- Faster $\rightarrow$ requires smaller features on chip

- Smaller $\rightarrow$ **greater** electrical resistance !
- $\Longrightarrow\Longleftarrow$.
- Serial processors have hit a wall.



Some parallel programming tools

- OpenMP—
  - Shared memory, multiple CPU core model.
  - Good for moderate parallelism.
  - Easy to get started.
  - Options for protecting parallel writes:
    - Sum reduction: no overhead.
    - Atomic add and capture: small overhead.
    - Critical block: perhaps 100K instruction overhead.
  - Valid cost metric: real time used.
  - 2-thread programs perhaps slower than 1-thread.
- CUDA/Thrust—
  - Nvidia C++ template library for CUDA based on STL.
  - Functional paradigm: easier algorithm expression.
  - Hides many CUDA details: good and bad.
  - Powerful operators all parallelize: scatter/gather, reduction by key, permutation, sort, prefix sum.
  - Surprisingly efficient algorithms like bucket sort.
  - Possible back ends: CUDA, OpenMP, sequential on host.

Geometric Databases

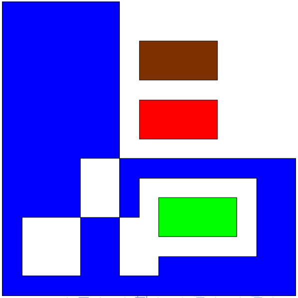
- Hundreds of millions of primitive components.
- Some foundational operations—
  - nearest point
  - boolean intersection and union
  - planar graph overlay
  - mass property computation of the results of some boolean operation
- Higher applications—
  - Volume and moments of an object defined as the union of many overlapping primitives.
  - Two object interfere iff volume of intersection > 0.
  - Interpolate population from census tracts to flood zones.
  - Interpolate properties between two triangulations of same polyhedron.
  - $\cdots$ and many higher-level problems.

How few types of info does a polyhedron rep need?

A design is not complete until everything possible has been removed.

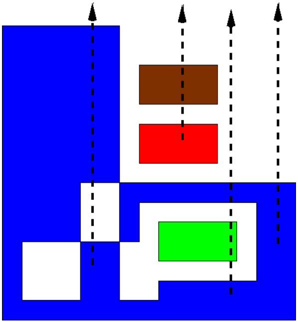
- Why?
  - fewer special cases $\Rightarrow$ less code $\Rightarrow$ less debugging
  - less space $\Rightarrow$ faster
- Operations:
  - point location
  - area, center of gravity, high-order moments

- Ambiguous rep: set of vertices.
- Sufficient rep: set of faces.
- Above operations are now map-reductions.



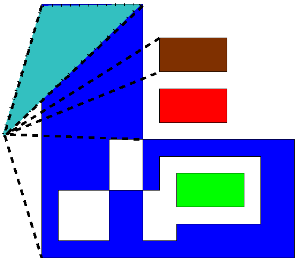
Point Location on a Set of Faces

- "Jordan curve" method
- Extend a semi-infinite ray from query point.
- Count intersections with faces.
- Odd number $\equiv$ inside.
- *Obvious but bad alternative: sum subtended signed volumes. Implementing w/o arctan, and handling special cases wrapping around is tricky and reduces to Jordan curve.*



Moment Computation on a Set of Faces

- Each face, with the origin, defines a tetrahedron.
- Compute its moment; sum them.
- Extends to any mass property, including (using a characteristic function) point location.
- Extends to functionally graded properties, e.g., 3D printer extruding a varying-density material.



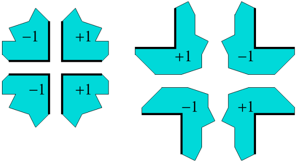
The Advantages of Set of Faces Data Structure

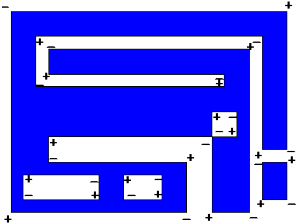
- Simple enough to debug.
- *SW can be simple enough that there are obviously no errors, or complex enough that there are no obvious errors.*
- Less storage.
- Easy parallelization: reduction operations.

$\exists$ Other reps (on the following slides).

Augmented vertices: another minimal polyhedron representation

- Augmented vertices: add a little to each vertex.
- These examples use rectilinear polygons, but all this works on general polygons and polyhedra.
- 8 types of vertices;
- Each gets a sign, $s = \pm 1$.
- Now, each vertex defined as $v_i = (x_i, y_i, s_i)$
- Area of polygon: $A = \sum s_i x_i y_i$
- Volume of polyhedron: $V = \sum s_i x_i y_i z_i$



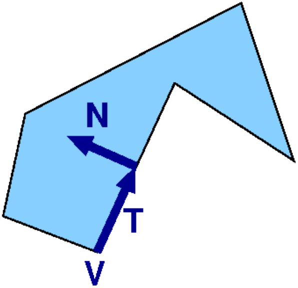


- Moment of inertia about z-axis: $I = \sum s_i x_i^2 y_i^2$

Vertex incidences: YAMPR

Another minimal data structure, resembles half edges.

- Only data type is the incidence of an edge and a vertex, plus its neighborhood. For each such:
  - $\vec{V}$ = coord of vertex
  - $\hat{T}$ = unit tangent vector along the edge
  - $\hat{N}$ = unit vector normal to $\hat{T}$ pointing into the polygon.
- Polygon (2 tuples per vertex): $\{(\vec{V}, \hat{T}, \hat{N})\}$
- Perimeter $= -\sum(\vec{V} \cdot \hat{T})$.
- Area $= 1/2 \sum(\vec{V} \cdot \hat{T})(\vec{V} \cdot \hat{N})$
- Multiple nested components ok.



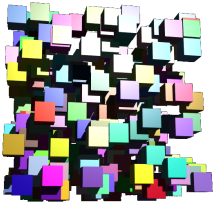
- Mass properties are map-reductions.

What's the point of this?

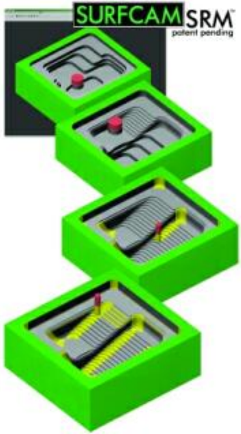
- Don't we always know the edges?
- No, not easily for Boolean combinations.
- We know the input polyhedra's faces.
- However finding the output polyhedron's faces is much harder than merely finding the augmented vertices.
  - That requires finding more global topology.
- Three types of output vertices—
  - Some input vertices,
  - Some intersections of three input faces.
  - Some intersections of and input face with an edge.
- Filter them: an output vertex must be—
  - for intersection: *inside* all input polyhedra.
  - for union: *outside* all input polyhedra.
- Apply reduction equation to surviving vertices.
- *Next:* several examples.

Volume of Union of Many Cubes

- Illustrates power of these ideas.
- A prototype on an easy subcase (congruent axis-aligned cubes).
- Extends to general polyhedra.
- *Not* statistical sampling—this is exact output, apart from roundoff.
- *Not* subdivision-into-voxel method — the cubes' coordinates can be any representable numbers.

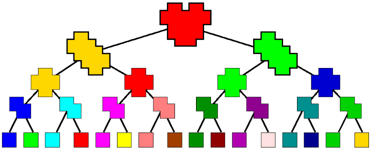


Application: Cutting Tool Path



- Represent path of a tool as piecewise line.
- Each piece sweeps a polyhedron.
- Volume of material removed is (approx) volume of union of those polyhedra.
- Image is from Surfware Inc's Surfcam website.

Traditional N-Polyhedron Union



- Construct pairwise unions of primitives.
- Iterate.

Time depends on intermediate swell, and elementary intersection time.

- Let P = size of union of an M-gon and an N-gon. Then P=O(MN).
- Time for union (using line sweep) $T = \Theta(P \lg P)$.
- Total $T = O(N^2 \lg N)$.

Hard to parallelize upper levels of computation tree.

Problems With Traditional Method

- $\lg N$ levels in computation tree cause $\lg N$ factor in execution time. Consider $N > 20$.
- *Intermediate swell:* worse as overlap is worse. Intermediate computations may be much larger than final result.
- The explicit output polyhedron has complicated topology: unknown genus, loops of edges, shells of faces, nonmanifold adjacancies.
- Tricky to get all this right.
- *However* explicit output not needed for computing mass properties.
- Set of vertices with neighborhoods suffices.

Fast parallel volume of union

- Find the intersections in one flat intersection test.
- Filter them.
- Map-reduce them.
- Processing 100M cubes with $L = 0.005$ using 1000^3 grid took 5800 secs.
- Computed 3M face–edge and 3M face–face–face intersections.
- Optimization: many grid cells were completely inside an input cube.
- Note that this is not simply streaming the data—these are triple-object incidences.

2D and 3D overlay

2D planar graph

- Input: two planar graphs containing sets of polygons (aka faces).
- Output: all the nonempty intersections of one polygon from each map.
- Example: Census tracts with watershed polygons, to estimate population in each watershed.

3D triangulated polyhedra

- presented at International Meshing Roundtable 2018.

Important data structure

uniform grid.

Uniform grid

Summary

- Overlay a uniform 3D grid on the input.
- For each input primitive—face, edge, vertex—find overlapping cells.
- In each cell, store set of overlapping primitives.

Properties

- Simple, sparse, uses little memory if well programmed.
- Parallelizable.
- Robust against moderate data nonuniformities.
- Bad worst-case performance on extremely nonuniform data.
  - As do octree and all hierarchical methods.

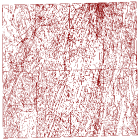
How it works

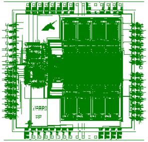
- Intersecting primitives must occupy the same cell.
- The grid filters the set of possible intersections.

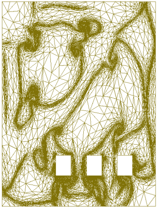
Uniform Grid Qualities

- Major disadvantages: It's so simple that it apparently cannot work, especially for nonuniform data. Efficient implementing takes care.
- Major advantage: For the operations I want to do (intersection, containment, etc), it works very well for any real data I've ever tried.
- Outside validation: used in our 2nd place finish in ACM 2016 SIGSPATIAL GIS Cup award.

USGS Digital Line Graph; VLSI Design; Mesh







2D Uniform Grid Time Analysis

For i.i.d. edges (line segments), time to find edge–edge intersections in E^2 is linear in size(input+output) regardless of varying number of edges per cell.

- N edges, length 1/L, $G \times G$ grid.
- Expected # intersections $= \Theta\left(N^2 L^{-2}\right)$.
- Each edge overlaps $\leq 2(G/L+1)$ cells.
- $\eta \triangleq$ # edges per cell, is Poisson; $\overline{\eta} = \Theta(N/G^2(G/L+1))$.
- Expected total # xsect tests: $G^2\overline{\eta^2} = N^2/G^2(G/L+1)^2$.
- Total time: insert edges into cells + test for intersections. $T = \Theta\left(N(G/L+1) + N^2/G^2(G/L+1)^2\right)$.
- Minimized when $G = \Theta(L)$, giving $T = \Theta\left(N + N^2 L^{-2}\right)$.
- $T = \Theta$ (size of input + size of output).

■

EPUG-Overlay: 2D planar graph overlay

- Previous step, presented at 2015 ACM BIGSPATIAL
- Biggest example: USWaterBodies: 21,652,410 vertices, 219,831 faces, with USBlockBoundaries: 32,762,740 vertices, 518,837 faces.
- Time (w/o I/O):
  - 1342 secs (1 thread);
  - 149 secs (16 cores, 32 threads).
  - 9X parallel speedup.

PINMESH: 3D point location

- Previous step, presented at 2016 Berlin Geometry Summit
- Uses rational numbers, Simulation of Simplicity, uniform grid, parallelism, simple data structures
- Biggest example: sample dataset with 50 million triangles.
  - Preprocessing: 14 elapsed seconds on 16-core Xeon.
  - Query time: 0.6 s per point.

Exact fast parallel intersection of large 3-D triangular meshes

- Intersect 3D meshes while
- Handling geometric degeneracies, including
  - Mesh with itself,
  - Mesh with its translation,
  - Mesh with its rotation.
- With no roundoff errors.
- Fast in parallel.
- Economical of memory.
- Extensively tested on hard cases.
- Compared to competing implementations.
- Example: Intersection of two big meshes from AIM@SHAPE: Ramesses: 1.7 million triangles x Neptune: 4 million triangles. 5.5 seconds on multicore Xeon.

Five key techniques

- Arbitrary precision rational numbers: for exactness.
- Simulation of Simplicity: for ensuring all the special cases are properly handled.
- Simple data representation and local information: parallelization and correctness.
- Uniform grid: accelerate computation; quickly constructed in parallel.
- Parallel programming

Hard part: making everything fit together.

Summary

The following techniques don't solve all the world's problems, but handle some foundational geometric ones nicely:

- deprecate hierarchies—
  - simple geometric representation,
  - bucket sort objects with uniform grid.
- local server HW processes large datasets in parallel.
- handles inter-object coincidences (not just streaming processing).
- exploits synergy between CAD and GIS.