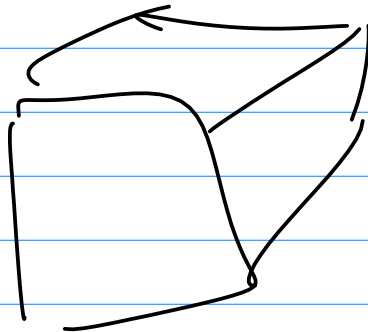


q: in viewport, what model space coordinates map to the whole viewport?

a: set by glOrtho or glFrustum

visible human project?

chapter 4: transformations



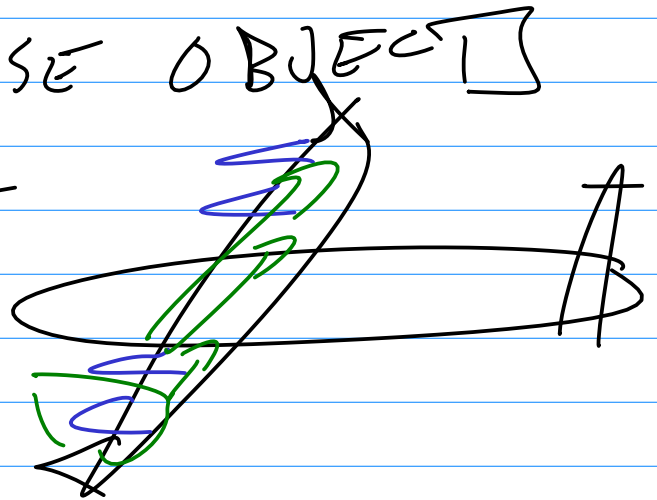
ROTATE

TRANSLATE

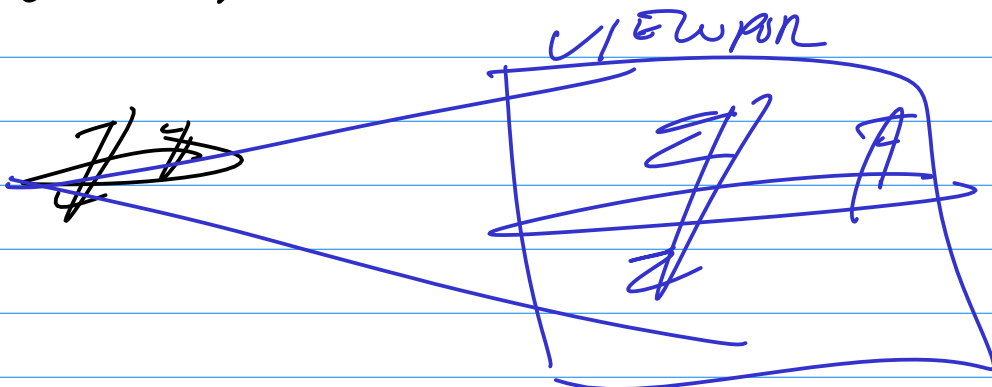
SCALE

WHY? REUSE OBJECTS

DESIGN AIRPLANE
4 IDENTICAL ENGINES



SCALE - MAKE IMAGE FIT



All common transformations will be a 4x4 matrix multiplication.

- only 1 piece of code

translate, rotate, scale, perspective projection

- you can combine the matrices into one matrix before applying it.

- fast

Modelview Matrix

Translate by (dx, dy, dz)

Point (x,y,z):

$$P = \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & 0 & 0 & D_x \\ 0 & 1 & 0 & D_y \\ 0 & 0 & 1 & D_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P' = MP = \begin{pmatrix} 1 & 0 & 0 & D_x \\ 0 & 1 & 0 & D_y \\ 0 & 0 & 1 & D_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} X + D_x \\ Y + D_y \\ Z + D_z \\ 1 \end{pmatrix}$$

~~Ex~~ $P = (1, 2, 3)$ TRANS: $(5, 6, 8)$
 TRANSLATE POINT $(1, 2, 3)$ BY
 DISTANCE $(5, 6, 8)$

$$\begin{pmatrix} 6 \\ 8 \\ 11 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$$

scale: uniform or nonuniform

UNIFORM SCALE BIS

$$\begin{pmatrix} S & 0 & 0 & 0 \\ 0 & S & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

SCALE (1, 2, 3) BY 10

$$\begin{pmatrix} 10 \\ 20 \\ 30 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$$

NONUNIFORM SCALE: S_x, S_y, S_z

$$\begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4

SCALE BY 2 IN X, 3 IN Y, 4 IN Z

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

SCALE BY \Rightarrow ; REFLECTION

REFLECT IN X AXIS

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

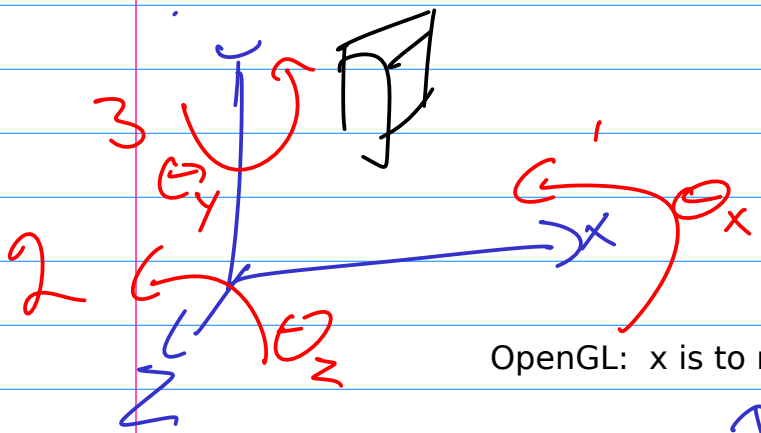
WHEN WOULD YOU REFLECT A PART?
BICYCLE PEDAL

2 PEDALS
MIRROR IMAGES.

Rotation in 3D as a matrix multiply

1st q: what's the API? IOW how do we want to specify the rotation?
 There're at least 3 common ways.

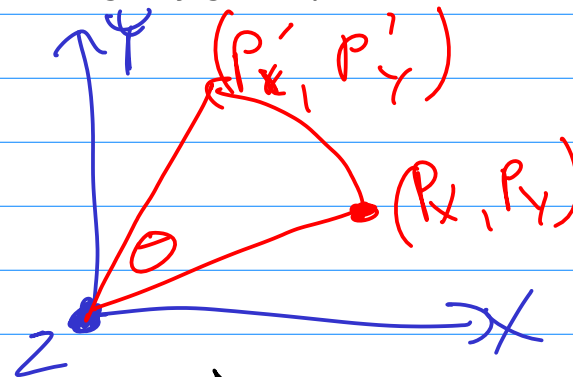
a1: Euler angles



new q: how to rotate around an axis?
 a: it's really a 2D rotation.

OpenGL: x is to right, y goes up, z comes towards me.

Rotate about z axis:



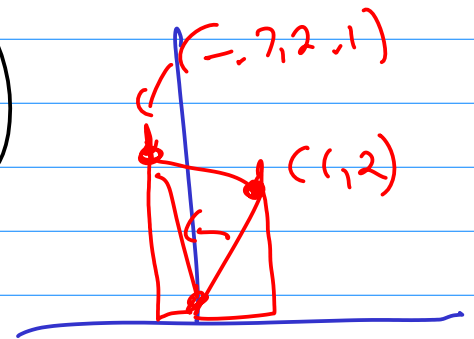
rule:

$$\begin{pmatrix} P_x' \\ P_y' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} P_x \\ P_y \end{pmatrix}$$

2x2

$\theta = 45^\circ = \pi/4$ $\cos \theta = \sin \theta = \frac{1}{\sqrt{2}} = .7$

$$\begin{pmatrix} -.7 \\ 2.1 \end{pmatrix} = \begin{pmatrix} .7 & -.7 \\ -.7 & .7 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$



how to make that 2D rotation into a 3D rotation about z axis?

2D: $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ 3×3 $\begin{pmatrix} \cos \theta_z & -\sin \theta_z & 0 & 0 \\ \sin \theta_z & \cos \theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Z won't change
points on axis don't change

X AXIS. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x & 0 \\ 0 & \sin \theta_x & \cos \theta_x & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

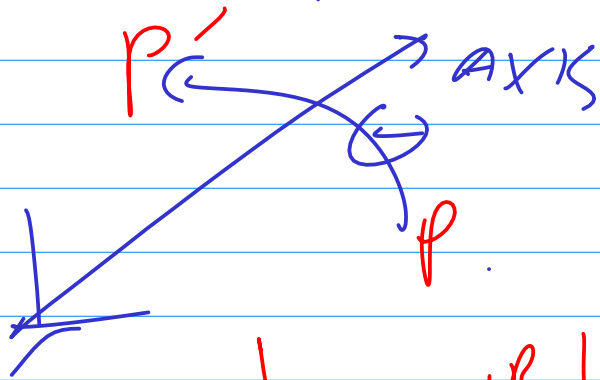
Y AXIS $\begin{pmatrix} \cos \theta_y & 0 & \sin \theta_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta_y & 0 & \cos \theta_y & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

OR. $SI\ ROTATE$ ()

$R_z R_y R_x P$
 R_p

NEXT TIME
ART

AXIS, ANGLE



↓ MATRIX

→ QUATERNIONS

BACK: MATRIX → AXIS, ANGLE

I HAVE
WEB
PAGES