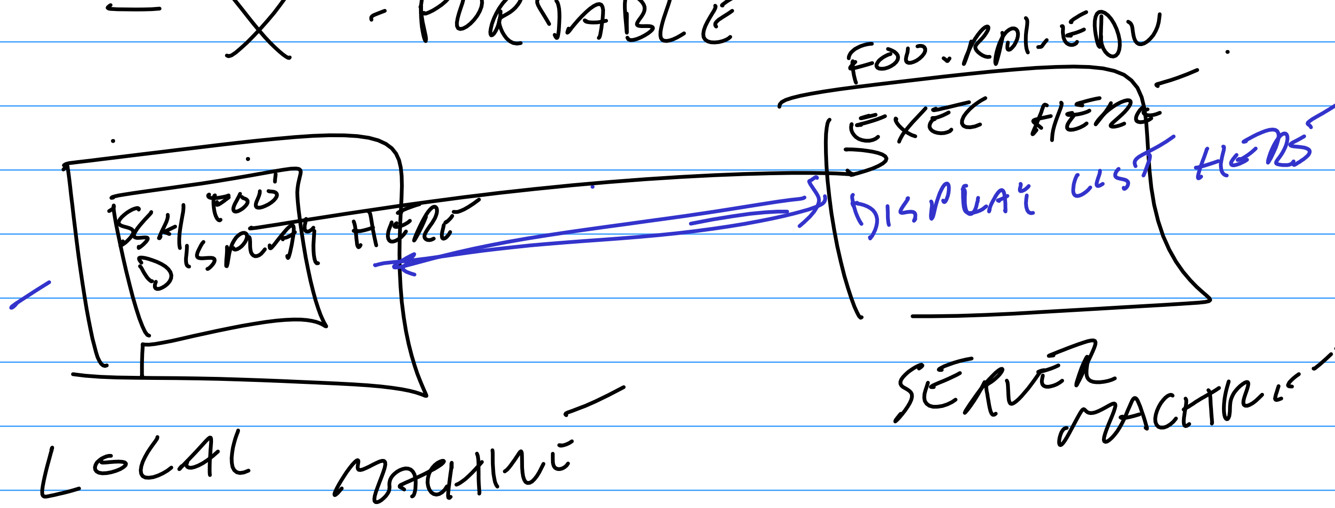
REMOTE GRAPHICS
- X - PORTABLE



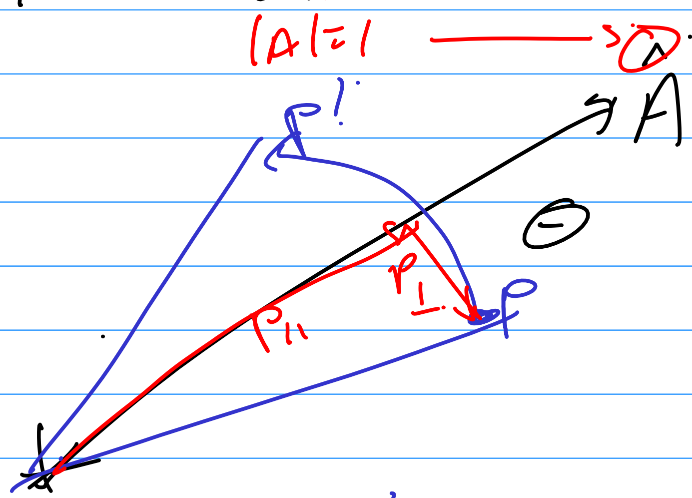
LINUX

$ time prog args

how to measure time your program takes

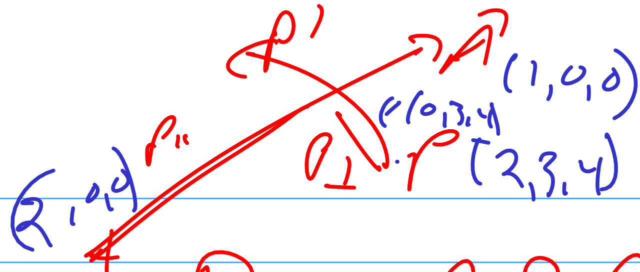
1. time command at shell
2. time subroutine calls in program.

ROTATION BY AXIS + ANGLE



POINTS ON AXIS DON'T MOVE

$$P = P_{\parallel} + P_{\perp}$$



$|A| = 1$

$$P_\parallel = A \cdot P\, A$$

$$P_\perp = P - P_\parallel$$

eg $A = (1,0,0)$

$P = (2,3,4)$

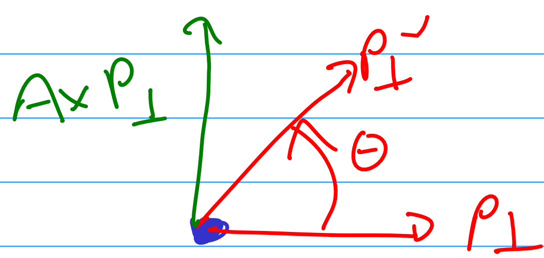
$$A \cdot P = (1,0,0) \cdot (2,3,4) = 2$$

$$P_\parallel = A \cdot P\, A = 2 \cdot (1,0,0) = (2,0,0)$$

$$P_\perp = P - P_\parallel = (2,3,4) - (2,0,0) = (0,3,4)$$

$$P_\parallel' = \text{ROTATED } P_\parallel = P_\parallel = (2,0,0)$$

$P_\perp$ ROTATES IN A 2D SYSTEM



A COMING STRAIGHT OUT OF SCREEN

$$P_\perp' = \cos\theta\ P_\perp + \sin\theta\ A \times P_\perp$$

eg. $A \times P_\perp = (1,0,0) \times (0,3,4) = (0,-4,0)$

$$P' = P_\parallel' + P_\perp'$$

$$P' = P_{\parallel}' + P_{\perp}'$$

$$= P_{\parallel} + \cos\theta \, P_{\perp} + \sin\theta \, A \times P_{\perp}$$

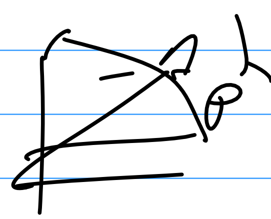
$P_{\parallel} = (A \cdot P) A \qquad P_{\perp} = P - A \cdot P A$

$$P' = A \cdot P A + \cos\theta \, (P - A \cdot P A) + \sin\theta \, A \times (P - A \cdot P A)$$

$$= A \cdot P A + \cos\theta \, (P - A \cdot P A) + \sin\theta \, A \times P \qquad A \times A = 0$$

$$\boxed{P' = \cos\theta \, P + (1 - \cos\theta) A \cdot P A + \sin\theta \, A \times P}$$

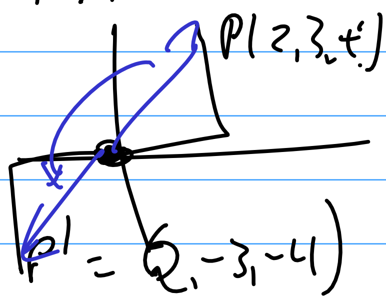
1. $\theta = 0$; $P' = P + 0 + 0 = P$
2. $P > A$. $P' = \cos\theta \, P + (1 - \cos\theta) A = P$
3. $\theta = 180°$ $\quad \cos\theta = -1$



$P' = -P + 2 A \cdot P A$

ej. $P = (2,3,4)$ $\quad A = (1,0,0)$ $\quad A \cdot P = 2$ $\quad A \times P = (0,-4,0)$

$\theta = 180°$ $\quad P' = (-2,-3,-4) + 4(1,0,0) = (2,-3,-4)$



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PROBLEM

THAT VECTOR FORMULA IS NOT A MATRIX MULTIPLICATION.

I WANT A MATRIX M.

M DEPENDS ON A, θ $= M_{A,\theta}$

$$P' = M \cdot P$$

$$\boxed{P' = \cos\theta\, P + (1-\cos\theta) A \cdot P\, A + \sin\theta\, A \times P}$$

$$\cos\theta\, P = M_1 P$$

$$M_1 = \begin{pmatrix} \cos\theta & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & \cos\theta \end{pmatrix}$$

I WANT $M_2 \ni$

$$M_2 P = (1-\cos\theta) A \cdot P\, A$$

$$A \cdot P\, A = \begin{pmatrix} a_1^2 & a_1 a_2 & a_1 a_3 \\ a_1 a_2 & a_2^2 & a_2 a_3 \\ a_1 a_3 & a_2 a_3 & a_3^2 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

$$(a_1 p_1 + a_2 p_2 + a_3 p_3) a = \begin{pmatrix} a_1^2 p_1 + a_1 a_2 p_2 + a_1 a_3 p_3 \\ a_1 a_2 p_1 + a_2^2 p_2 + a_2 a_3 p_3 \\ a_1 a_3 p_1 + a_2 a_3 p_3 + a_3^2 p_3 \end{pmatrix}$$

$$\begin{pmatrix} a_1^2 p_1 + a_1 a_2 p_2 + a_1 a_3 p_3 \\ \leftarrow \\ \leftarrow \end{pmatrix}$$

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I WANT MATRIX M_3

M_3 IS COMPUTED FROM A

$$A \times P = M_3 P$$

$$M_3 = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} -a_3 p_2 + a_2 p_3 \\ a_3 p_1 - a_1 p_3 \\ -a_2 p_1 + a_1 p_2 \end{pmatrix}$$

$$= a \times p$$

THAT WAS GOING FROM AXIS+ANGLE $\Rightarrow$ MATRIX.

WHAT IF: USER GIVES YOU A MATRIX TO ROTATE WITH.

Q: IS M_A ROTATION ?

L6

WHAT ARE PROPERTIES OF A ROTATION?

IT IS RIGID.

DISTANCES, ANGLES, VOLUMES DO NOT CHANGE.

IF I ROTATE $\begin{pmatrix}1\\0\\0\end{pmatrix}$, RESULT MUST HAVE LENGTH 1

$$\begin{pmatrix} m_{11} & m_{12} & \\ & \cdot & \\ & & \end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}m_{11}\\m_{21}\\m_{31}\end{pmatrix} = C_1 \qquad |C_1| = 1$$

$$\cancel{\begin{pmatrix}1&2&3\\4&5&6\\7&8&9\end{pmatrix}} \qquad C_1 = \begin{pmatrix}1\\4\\7\end{pmatrix} \qquad |C_1| = \sqrt{66} \neq 1$$

$$|C_2| = 1 \qquad |C_3| = 1$$

ANGLES DO NOT CHANGE

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \perp \begin{pmatrix}0\\1\\0\end{pmatrix} \qquad C_1 \cdot C_2 = 0$$

$$C_i \cdot C_j = \delta_{ij} = \begin{cases} 1 & \text{IF } i = j \\ 0 & \text{IF } i \neq j \end{cases}$$

1 MORE RULE

IS THIS A ROTATION: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$?

$$x' = x$$
$$y' = y$$
$$z' = -z$$

NO, IT'S A REFLECTION

DEFINITION

TEST $|M| = 1$

FOR A REFLECTION $|M| = -1$

SUMMARY TEST IF M IS A ROTATION MATRIX:

COLUMN #'S $\rightarrow$ $C_i \cdot C_j = \delta_{ij}$ $\leftarrow$ KRONECKER δ

DETERMINANT $\rightarrow$ $|M| = 1$

IF-AND-ONLY-IF

NECESSARY + SUFFICIENT

IN REAL WORLD, USING FLOATS, NEED A TOLERANCE!