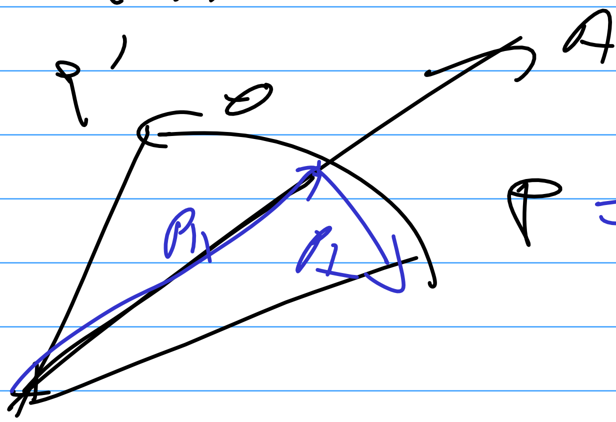
ROTATION REVIEW



$p = P_{\parallel} + P_{\perp}$

NICE FOR ANIMATION: STEP $\Delta\theta$

NOT FOR EULER ANGLES

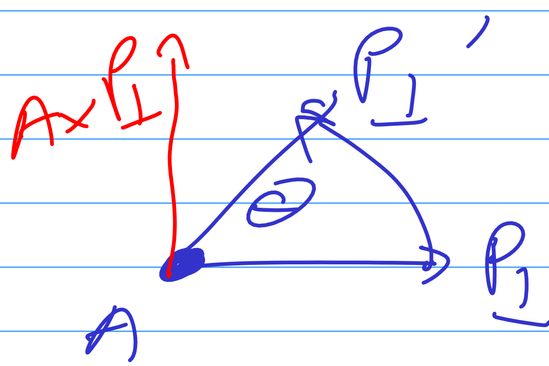
- SMOOTHLY CHANGING $\theta_x, \theta_y, \theta_z$

DOES NOT SMOOTHLY ROTATE OBJECT.

$P_{\parallel} = (A \cdot P)A$ IF $|A| = 1$

$P_{\perp} = P - P_{\parallel}$

$P_{\parallel}' = P_{\parallel}$



$P_{\perp}' = \cos\theta \, P_{\perp} + \sin\theta \, (A \times P_{\perp})$

$$A \cdot P \, A^{i} = M P$$

DEPENDS ON A
NOT ON P.

$$M_{ij} = a_i a_j$$

$$P' = M P$$

DEPENDS ON A, θ

ROTATION HAS 3 D.F.
2 FOR $\hat{A}$, 1 FOR θ

IS M A ROTATION?

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} m_{11} \\ m_{21} \\ m_{31} \end{pmatrix} \equiv C_1$$

LEN = 1

$$|C_1| = 1$$

$$C_1 \cdot C_2 = 0$$

$$\rightarrow C_i \cdot C_j = \delta_{ij}$$

$$|M| = 1$$

NEW

WHAT ARE AXIS + ANGLE?

EIGENVALUES - INVARIANT WHEN COORD SYSTEM CHANGES.

EIGENVECTORS

e.g. $M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ $\quad \lambda$ (EIGENVALUE) $= 1, 2, 3$

IF I ROTATE COORD SYSTEM

λ OF NEW MATRIX $= 1, 2, 3$

λ - EIGENVALUE

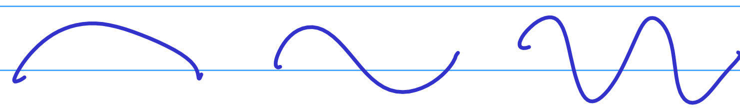
v - CORRESPONDING EIGEN VECTOR

$$Mv = \lambda v$$

3x3 MATRIX HAS 1-3 EIGENVECTORS + EIGENVALUES

EIGENVALUES ARE ROOTS OF A CUBIC EQUATION

$$|M - \lambda I| = 0$$

VIOLIN STRING v: 

M: ROTATION

WHAT ARE ITS E-VAL / VECT ?

ALL REAL EIGENVALUES ARE ± 1.

2 CASES $\lambda = 1, 1, 1$ IDENTITY M

OR $\lambda = 1, a \pm ib$

$\lambda = -1$ MEANS A REFLECTION ALONG CORRESPONDING E-VECT.

$a = \cos\theta$

$b = \sin\theta$

FROM E-VALS WE GET ANGLE OF ROTATION θ

AXIS? POINTS ON AXIS DON'T MOVE

I.E THEY ARE E-VECTS WITH E-VAL $\lambda = 1$.

M_1, M_2 - ROTATIONS

$M = M_1 M_2$

M_1, M_2 ARE RIGID TRANSFORMS

SO IS M

[M WILL BE A ROTATION WITH ITS AXIS + ANGLE]

SURPRISING!

EASIER WAYS TO FIND AXIS + ANGLE

DEF TRACE OF $M = m_{11} + m_{22} + m_{33}$

$Tr(M) = \sum \lambda_i =$

$1 + \cos\theta + i\sin\theta + \cos\theta - i\sin\theta$

$= 1 + 2\cos\theta$

$$\cos\theta = \frac{m_{11} + m_{22} + m_{33} - 1}{2}$$

YOU CAN COMBINE OFF-DIAGONAL ELEMENTS OF M TO GET AXIS - SEE WIKI

4D ROTATIONS ?

1ST DEFINITION.

RIGID LINEAR TRANSFORM

$$P' = M P$$

ROTATED POINT — 4x4 MATRIX — 4D POINT

RIGID $\Rightarrow$ DISTANCES DON'T CHANGE

ALL REAL E-VALS = 1

$$|M - \lambda I| = 0$$

3 CASES a) ALL $\lambda = 1$

b) λ: 1, 1, $\cos\theta \pm i\sin\theta$

2 COORDS FIXED, 2 ROTATION OF OTHER 2.

c) →

c) 2 2-D ROTATIONS

eg. x, y, z, t — x, y ROTATE

z, t ROTATE

THERE IS NO AXIS

EITHER 0 ONLY FIXED POINT

OR THERE IS A FIXED PLANE

APP OF 4-D ROTATION: SPECIAL RELATIVITY