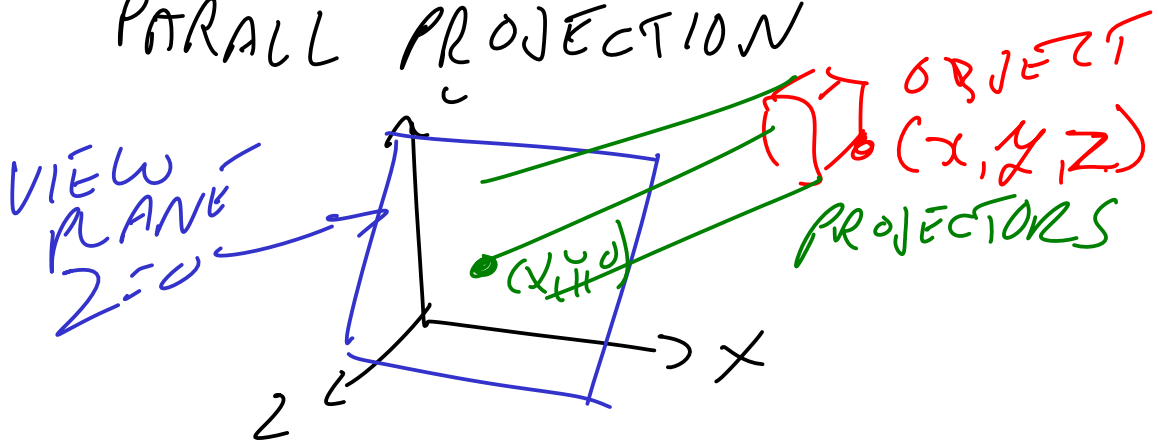
PROJECTION

ortho, frustum

PARALL PROJECTION



$(x,y,z) \rightarrow (x,y,0)$

3x3 MATRIX MULT

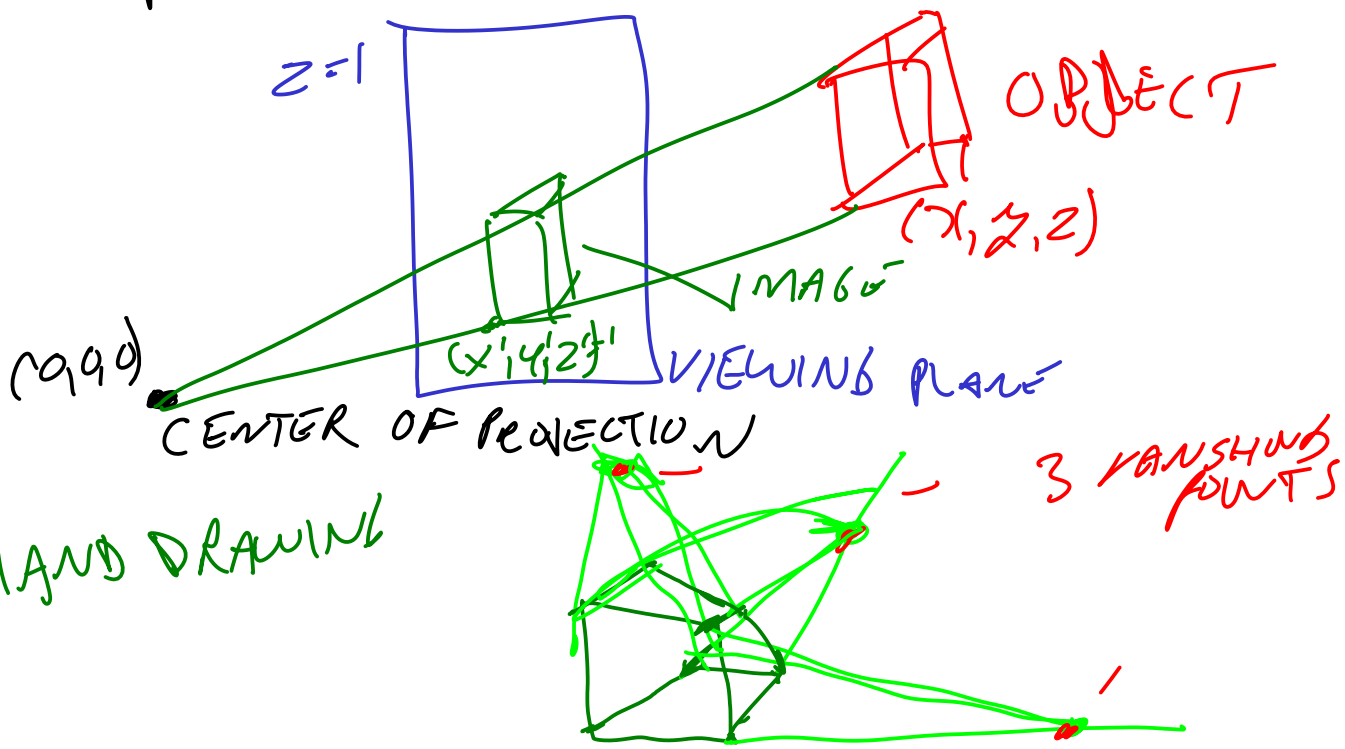
$$\begin{pmatrix}1\\2\\0\end{pmatrix} = \begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix}$$

4x4

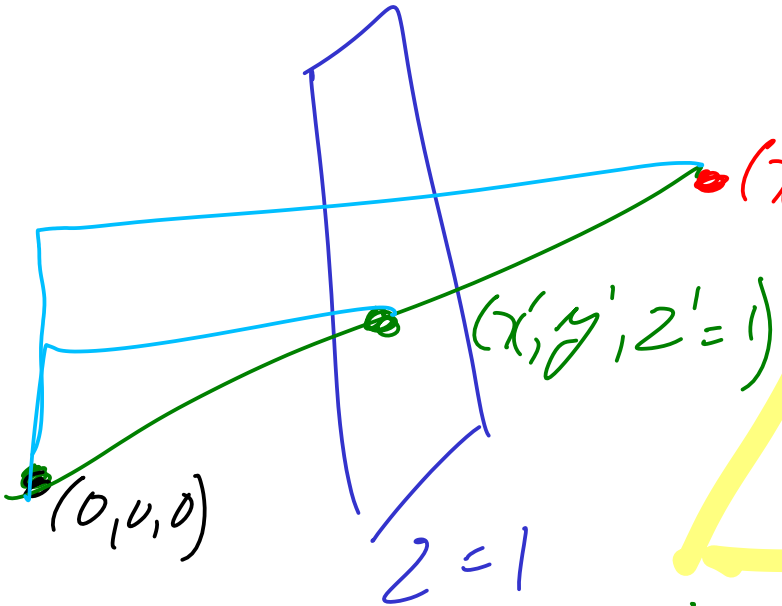
$$\begin{pmatrix}1\\2\\0\\1\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1\\2\\3\\1\end{pmatrix}$$

PERSPECTIVE PROJECTION

PERSPECTIVE PR 2



HAND DRAWING



$x' = \alpha x = \frac{x}{z}$

$y' = \alpha y = \frac{y}{z}$

$1 = z' = \alpha z = 1$

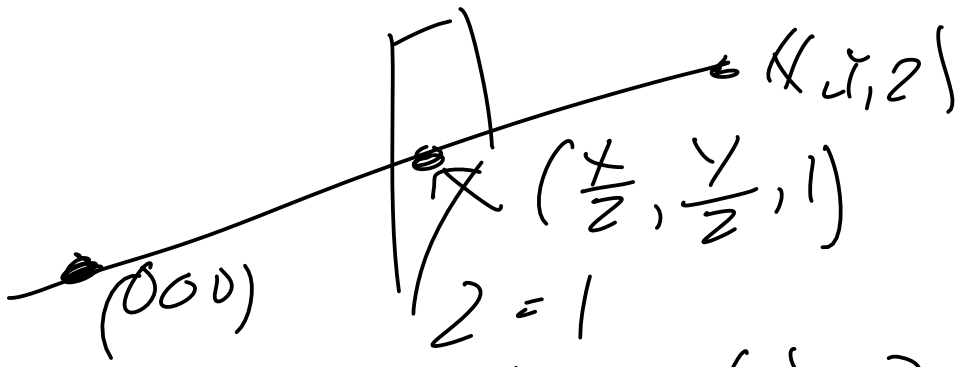
$\alpha = \frac{1}{z}$

SIMILAR TRIANGLES

X, Y, Z ALL SCALE THE SAME.

- FOR EACH POINT.

DIFFERENT POINTS SCALE DIFFERENTLY.



$(1,2,3) \Rightarrow \left(\frac{1}{3}, \frac{2}{3}, 1\right)$

I WANT TO MAKE THIS A MATRIX MULT
THIS NEEDS HOMOGENEOUS COORDS

3D H.C. HAS 4 NUMBERS.

OLD ONES: CARTESIAN: 3 NOS.

CART $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ H. $\begin{pmatrix} X_H \\ Y_H \\ Z_H \\ W \end{pmatrix} \rightarrow$ CART $\begin{pmatrix} X/W \\ Y/W \\ Z/W \end{pmatrix}$

↑ W = WEIGHT

H $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ C $\rightarrow \begin{pmatrix} 1/4 \\ 2/4 \\ 3/4 \end{pmatrix}$ H $\begin{pmatrix} 2 \\ 4 \\ 6 \\ 8 \end{pmatrix}$ C $\rightarrow \begin{pmatrix} 1/4 \\ 2/4 \\ 3/4 \end{pmatrix}$

$W \neq 0$ ($W = 0$; POINT AT ∞)

THOSE 4X4 TRANSFORM MATS ARE
H.C

TRANS $(1,2,3)$

$$\begin{pmatrix}20\\30\\40\\10\end{pmatrix}\begin{pmatrix}4\\6\\8\\2\end{pmatrix}\begin{pmatrix}2\\3\\4\\1\end{pmatrix}=\begin{pmatrix}1&0&0&1\\0&1&0&2\\0&0&1&3\\0&0&0&1\end{pmatrix}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\begin{pmatrix}2\\2\\2\\2\end{pmatrix}\begin{pmatrix}10\\10\\10\\10\end{pmatrix}$$

SAME CART PT $\begin{pmatrix}2\\3\\4\end{pmatrix}$

SAME CART PT $\begin{pmatrix}1\\1\\1\end{pmatrix}$

IT DOES NOT MATTER WHICH H.C. VERSION OF A CART PT YOU USE, WHEN YOU MULTIPLY BY THE MATRIX. THE H C RESULT ALWAYS GIVES SAME CART RESULT.

45° ROT ABOUT Z AXIS

$$\begin{pmatrix} 0 \\ 14 \\ 10 \\ 10 \end{pmatrix} \begin{pmatrix} 0 \\ 1.4 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} .7 & -.7 & 0 & 0 \\ .7 & .7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 10 \\ 10 \\ 10 \\ 10 \end{pmatrix}$$

WORKS FOR TRANSLATIONS, ROT < SCALE.

$$\begin{pmatrix} 2 \\ 3 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$x' \leftarrow x/z$

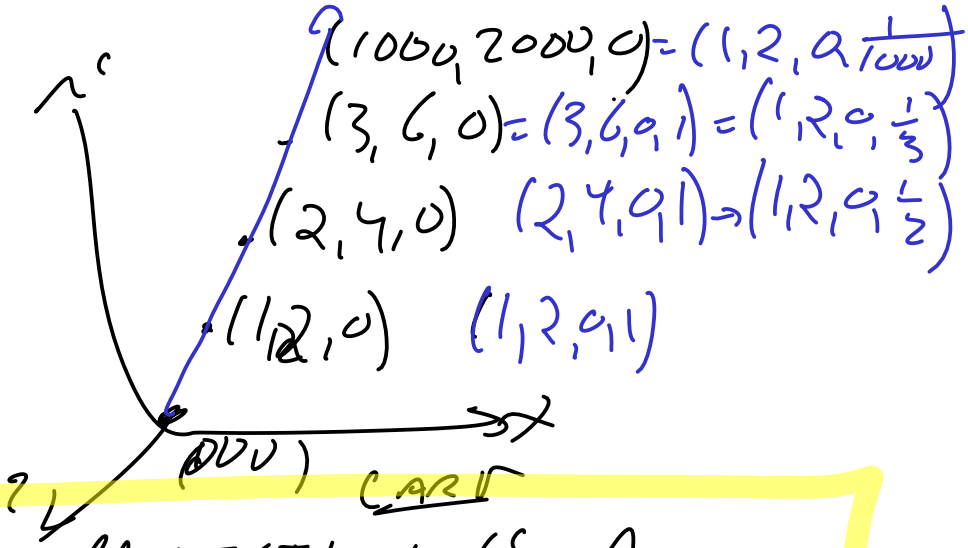
$y' \leftarrow y/z$

$z' \leftarrow 1$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

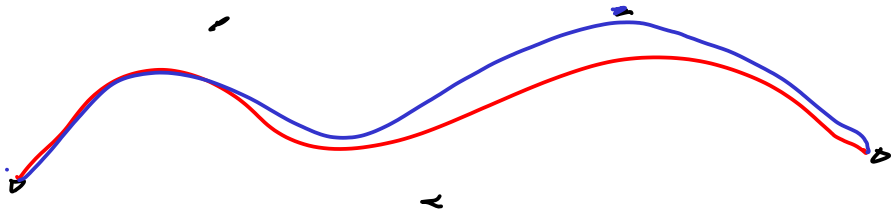
PERSPECTIVE PROJ IS JUST ANOTHER MATRIX MULT.

AT END OF LINE IS A POINT $(1,2,0,0)$



APP: A PARALLEL PROJECTION IS A PERSPECTIVE PROJECTION WITH CENTER AT ∞

GOOD API FOR DESIGNING CURVES IS TO SPEC CONTROL POINTS THE COMPUTER FITS A CURVE NEAR THEM



WITH HC, W IS A WEIGHT ATTRACTING CURVES

MORE PROJECTIONS

ORTHO CART.

$x' = x$
$y' = y$
$z' = 1$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad w = 1$$

$x' = x \quad z = 1$
$y' = y \quad w = 1$

PERS CART

$x' = x/z$
$y' = y/z$
$z' = 1$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

MOVE $w = 1$

$x' = x \quad z = z$
$y' = y \quad w = z$

CART $x' = x/w = x/z$

USEFUL TO UNDERSTAND THESE MATRICES.

ANOTHER PERSPECTIVE PIC.