

TRANSFORMATIONS

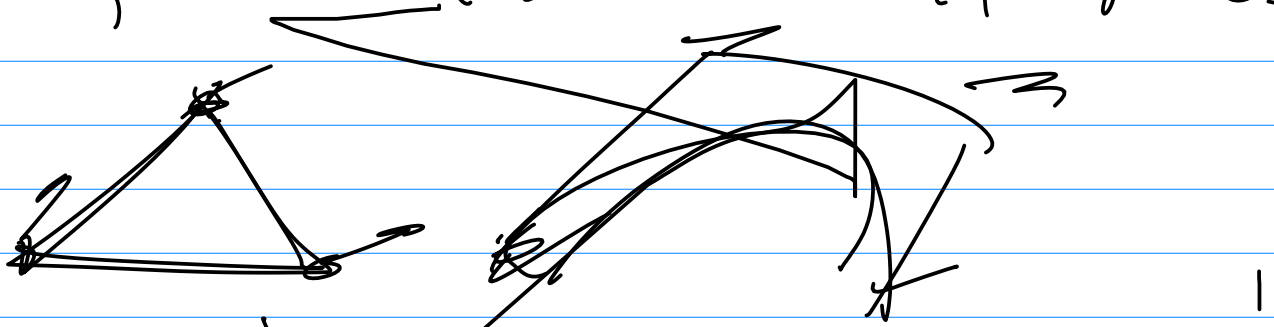
OBJECT POINTS

WHEN YOU

TRANSFORM THE

POINTS, THE REST OF IT FOLLOWS

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$



2D

$$P = \begin{pmatrix} X \\ Y \end{pmatrix}$$

SCALE BY S

MULTIPLY BY MATRIX

$$\begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} \rightarrow \begin{pmatrix} SX \\ SY \end{pmatrix}$$

ROTATE BY θ

$$R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

$$\theta = 45^\circ$$

$$R = \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \cdot \\ \cdot \end{pmatrix}$$

TRANSLATION?

WE WOULD LIKE TRANSLATION TO BE A MATRIX MULT.

THEN WE CAN COMBINE ALL THE MATRICES INTO 1 MATRIX.

TO DO THAT; USE HOMOGENEOUS COORDS

STILL 2D

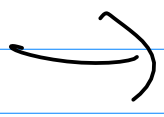
OLD: CART

NEW: HOM

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

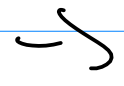
$$\begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix}_H$$



$$\begin{pmatrix} x/w \\ y/w \end{pmatrix}_C$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}_H$$



$$\begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix}_C$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}_C = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}_H, \quad \begin{pmatrix} 10 \\ 20 \\ 10 \end{pmatrix}_H,$$

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$$\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}_H$$

STILL 2D

CART SCALE

SCALE

$$S = \begin{pmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

JUST COPY IT

SCALE BY 3
CART $\begin{pmatrix} 30 \\ 03 \end{pmatrix}$

Apply $\begin{pmatrix} 30 & 0 \\ 03 & 0 \\ 00 & 1 \end{pmatrix}$

$$\begin{pmatrix} 6 \\ 15 \end{pmatrix} = \begin{pmatrix} 30 \\ 03 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 12 \\ 15 \\ 34 \end{pmatrix} \begin{pmatrix} 2 \\ 30 \\ 2 \end{pmatrix} = \begin{pmatrix} 30 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 6 \\ 23 \end{pmatrix}$$

45° CART ROT $\begin{pmatrix} .7 & -.7 \\ -.7 & .7 \end{pmatrix}$

Hom

$$\left(\begin{array}{cc|c} .7 & -.7 & 0 \\ -.7 & .7 & 0 \\ \hline 0 & 0 & 1 \end{array} \right)$$

CART

$$\begin{pmatrix} 2 \\ 12 \end{pmatrix} = \begin{pmatrix} .7 & -.7 \\ -.7 & .7 \end{pmatrix} \begin{pmatrix} 7 \\ 40 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 12 \end{pmatrix} = \begin{pmatrix} -.7 & -.7 & 0 \\ -.7 & .7 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 49 \\ 70 \\ 7 \end{pmatrix}$$

TRANSLATION

$$M = \begin{pmatrix} 1 & 0 & D_x \\ 0 & 1 & D_y \\ 0 & 0 & 1 \end{pmatrix}$$

TRANSLATE $B_1 \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

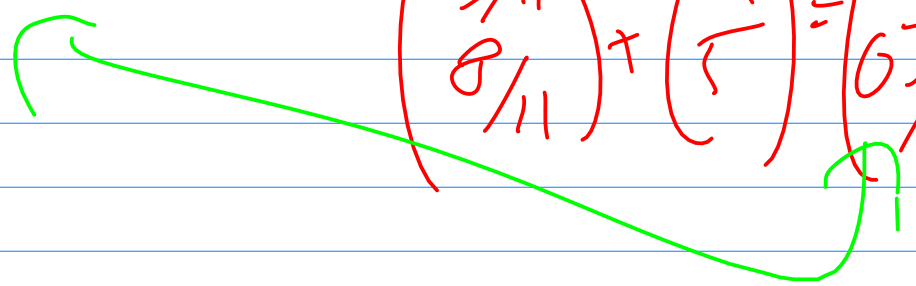
MULT B_1

$$\begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 8 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 8 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 25 \\ 63 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 8 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 25 \\ 63 \\ 11 \end{pmatrix}$$



YOU WANT TO DO THIS
TO OBJECT

1st ROT 30° M_1

2nd SCALE 5 M_2

3rd TRANS $(\frac{1}{2})$ M_3

4th ROT -10° M_4

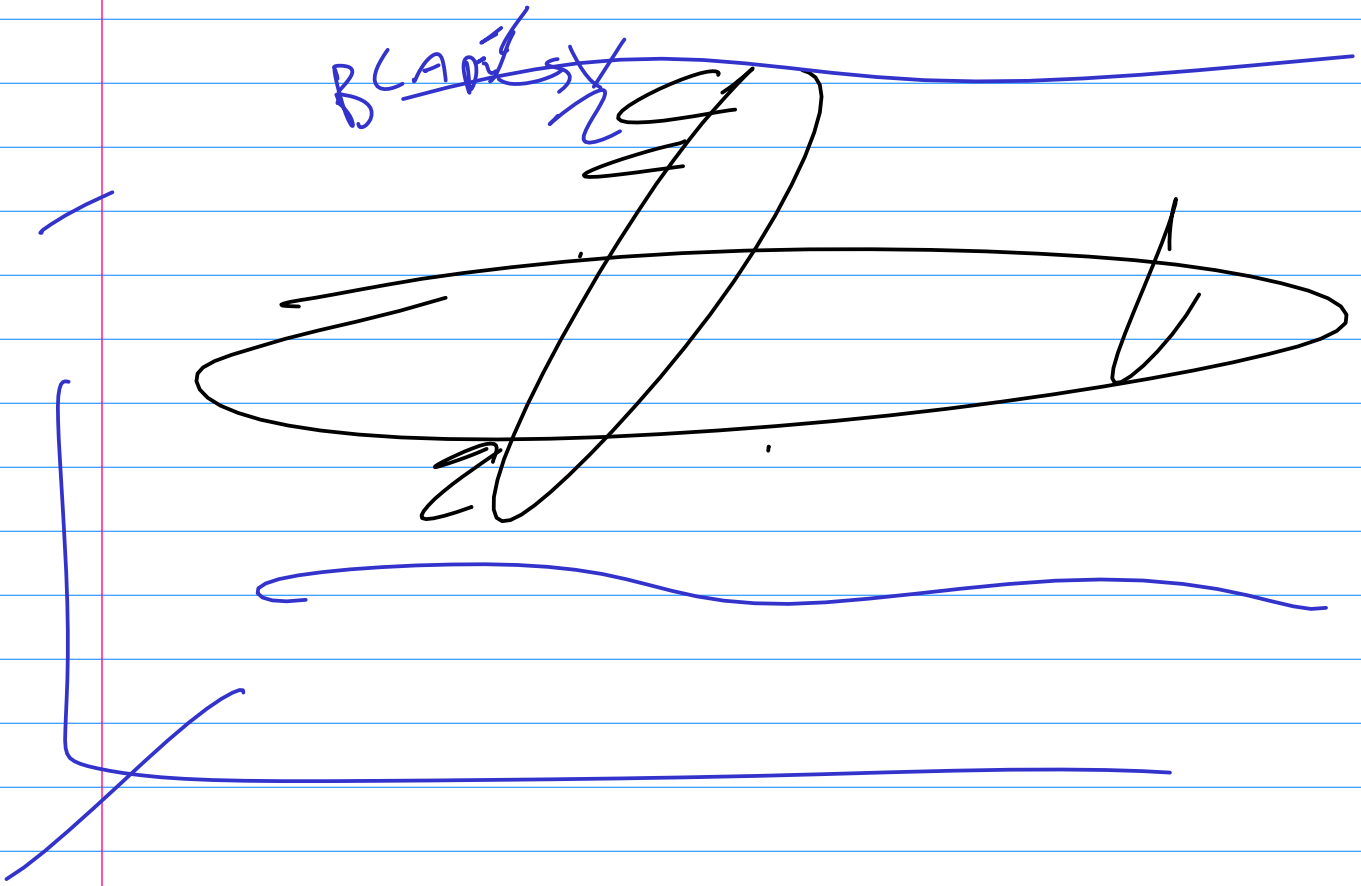
5th SCALE 2 M_5

$$M = M_5 M_4 M_3 M_2 M_1$$

$$p' = Mp$$

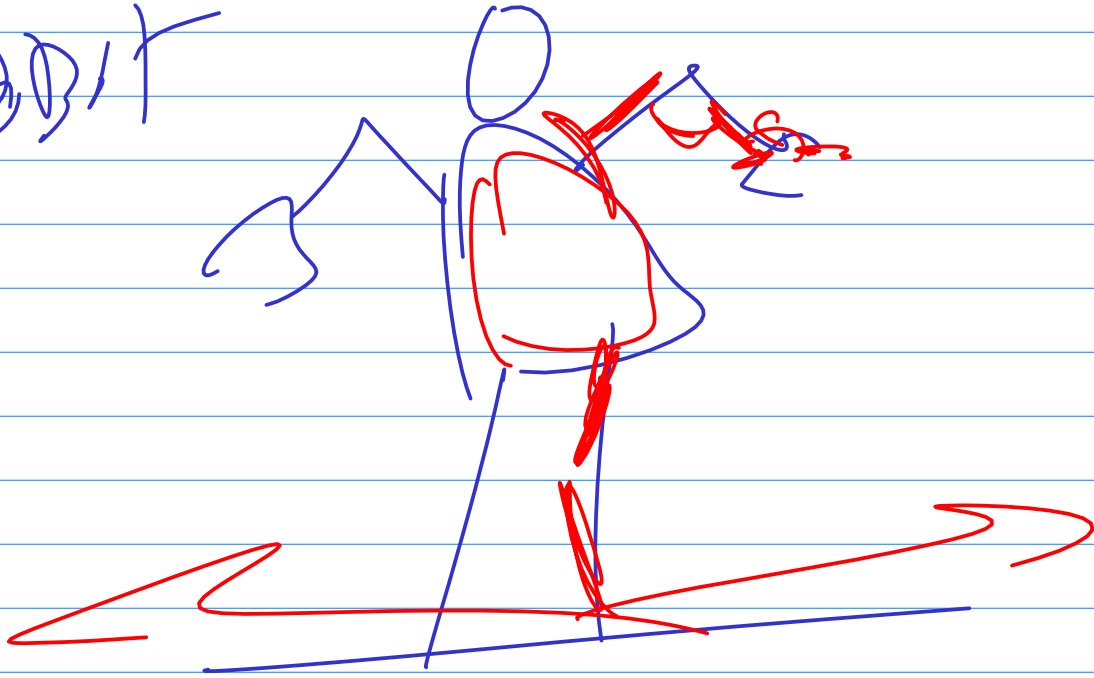
Q: WHY WOULD YOU
HAVE A SEQUENCE OF
TRANSFORMATIONS?

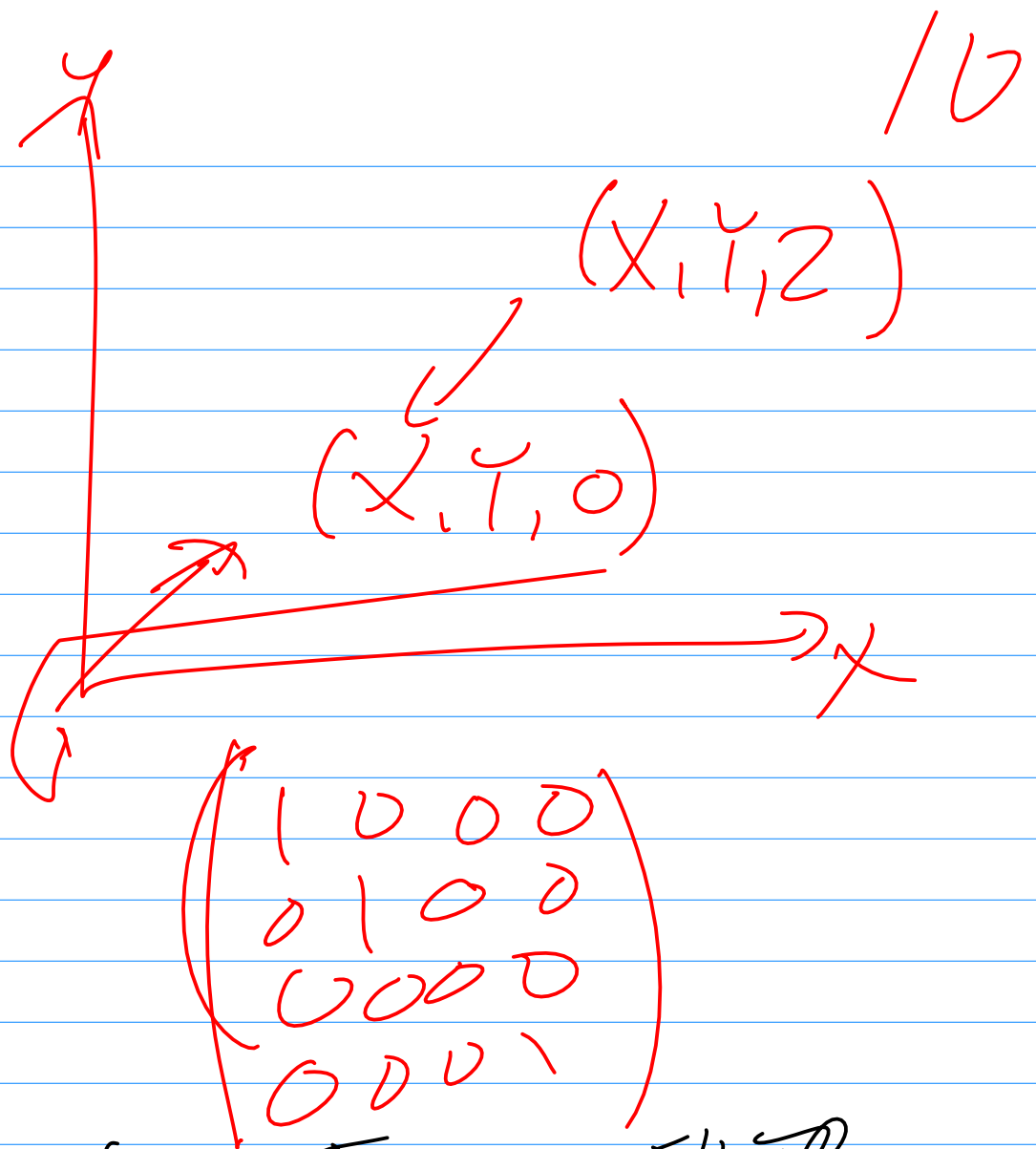
A: NESTED / HIERARCHICAL
COORDINATES



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Robot





IF YOU WANT ANOTHER
 PROJECTION -
 DISTORT THE OBJECT
 SO YOU CAN USE THAT
 PROJECTION

3D
Hom PT

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

CART

$$\begin{pmatrix} x/w \\ y/w \\ z/w \end{pmatrix}$$

LT $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ \rightarrow SC $\begin{pmatrix} 1/4 \\ 2/4 \\ 3/4 \end{pmatrix}$

SCALE

$$\begin{pmatrix} S & 0 & 0 & 0 \\ 0 & S & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

TR

$$\begin{pmatrix} 1 & 0 & 0 & 0x \\ 0 & 1 & 0 & 0y \\ 0 & 0 & 1 & 0z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3rd ROT

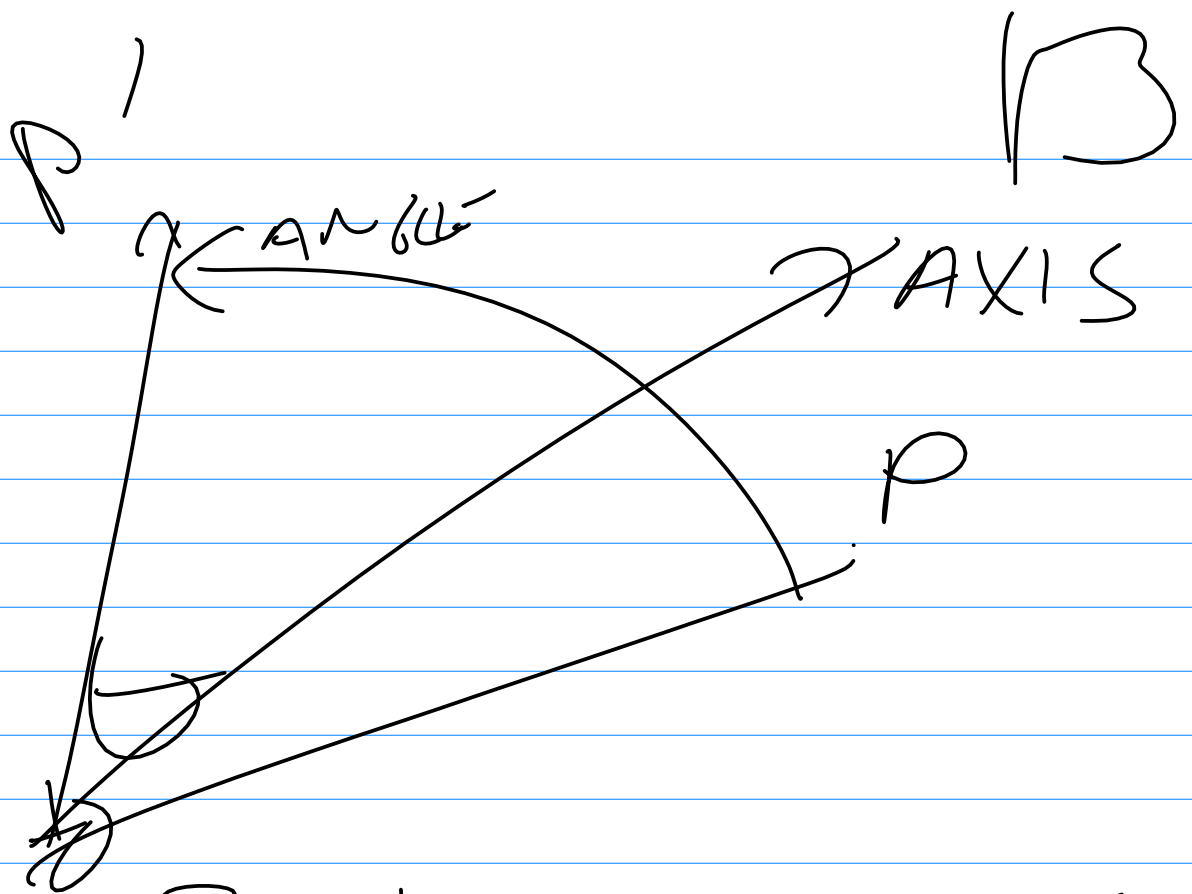
$$\left(\begin{array}{ccc|c} \cancel{3\pi/3} & & & 0 \\ \cancel{3\pi/3} & & & 0 \\ \hline 0 & 0 & 0 & 1 \end{array} \right)$$

R

Q HOW SHOULD USER DESCRIBE ROTATION?

A FIRST EULER ANGLES
ROT ABOUT X-AXIS
THEN Y
FINALLY Z

NICE FOR FIRST E
BAD FOR EVERYONE ELSE



ROTATE P BY θ ABOUT AXIS

PROBLEM: WHAT'S MATRIX?

PROB₂: GIVEN MATRIX,
A: IS IT A ROTATION?
B: WHAT'S AN ANGLE?

QUATERNIONS