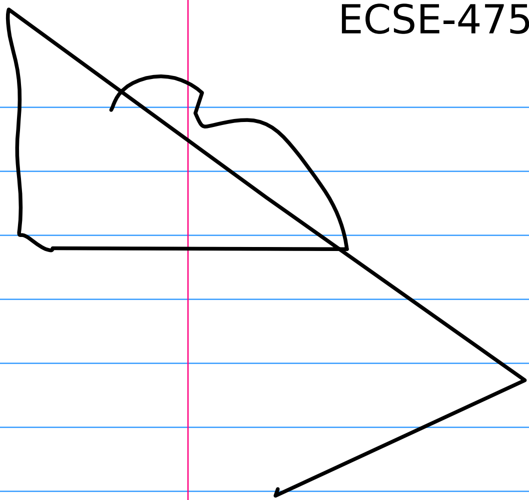
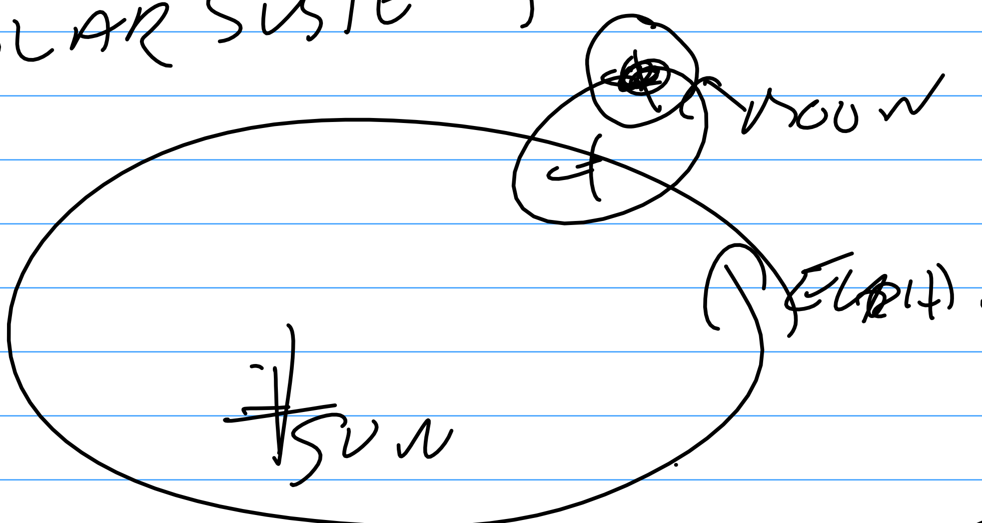


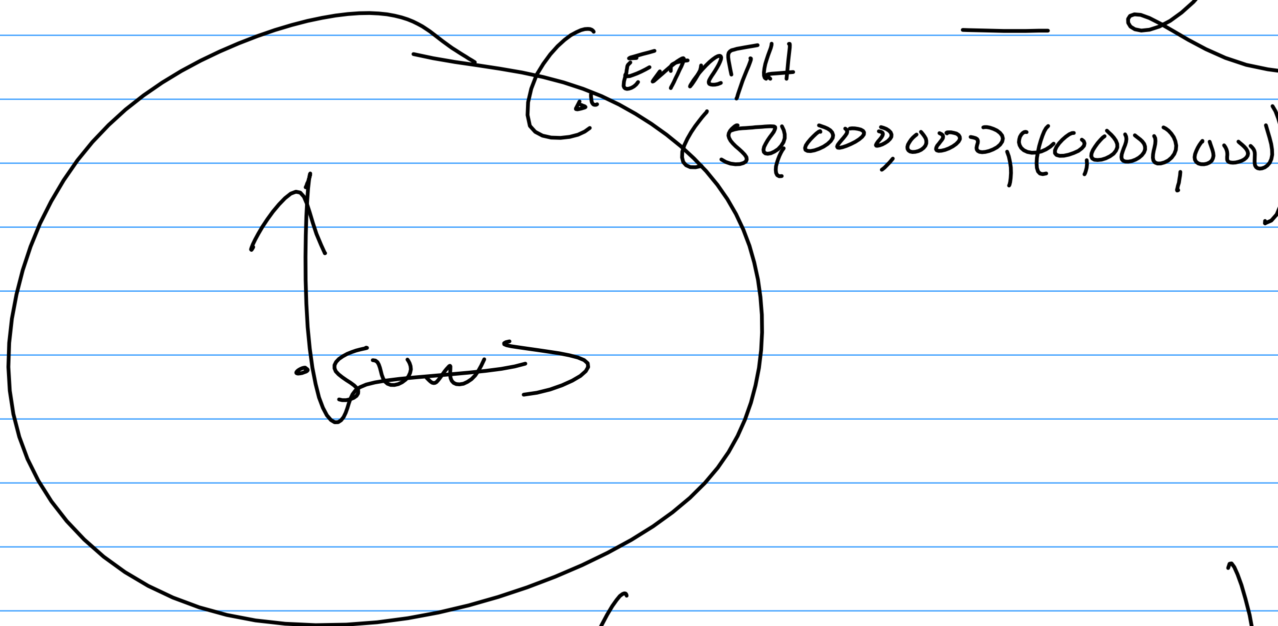
Q: HOW SHOULD THE USER DESCRIBE A ROTATION ABOUT ORIGIN?

CENTER OF ROTATION

SOLAR SYSTEM



1. TRANSLATE CENTER OF ROTATION TO THE ORIGIN.
2. ROTATE.
3. TRANSLATE BACK



1. T BY (−50M, −40M)
2. ROTAT.
3. T BY (+50M, +40M)

TO TRANSLATE A HOMOG POINT EITHER USE MATRIX OR CONVERT TO CART.

$P_H = (1, 2, 3)$ $\quad P_C = \left(\frac{1}{3}, \frac{2}{3}\right)$

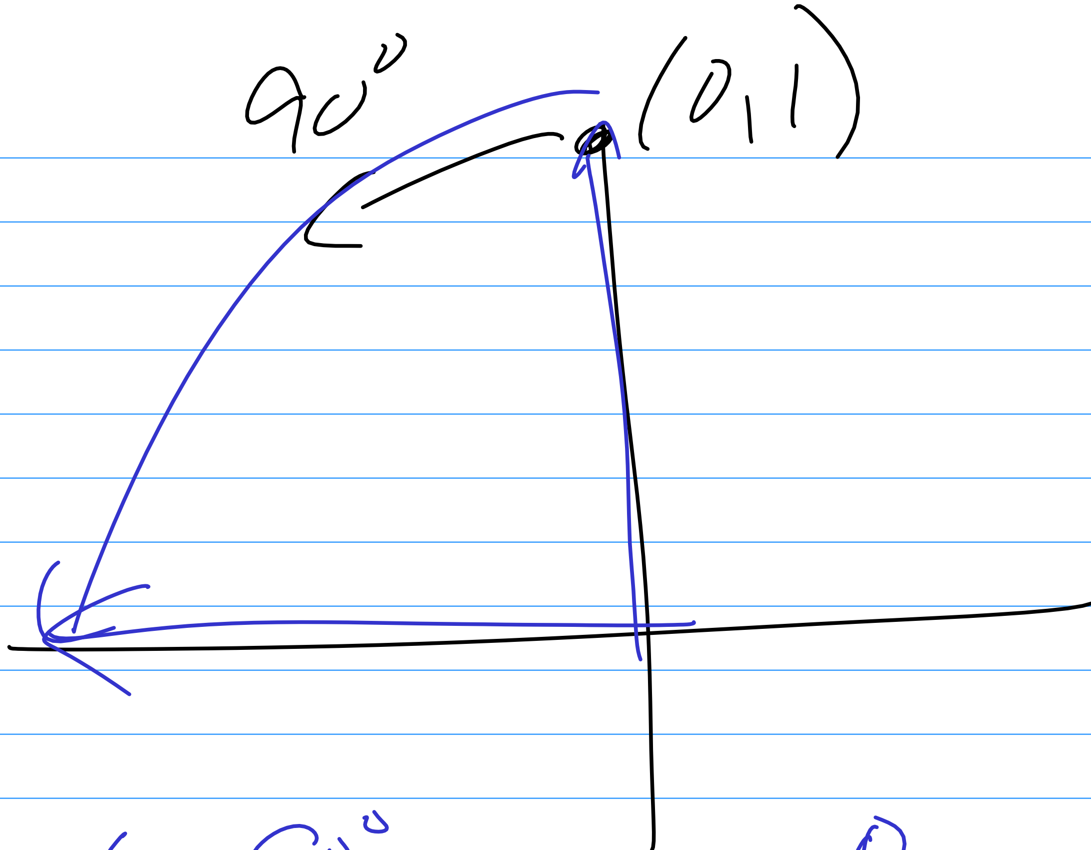
TRAN BY $D_x = 1$ $\quad D_y = 2$

$$P'_C = \left(\frac{1}{3}+1, \frac{2}{3}+2\right) = \left(\frac{4}{3}, \frac{8}{3}\right)$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 3 \end{pmatrix}$$

3

YOU CANNOT ADD HOMOG POINTS WITH DIFFERENT WTS.

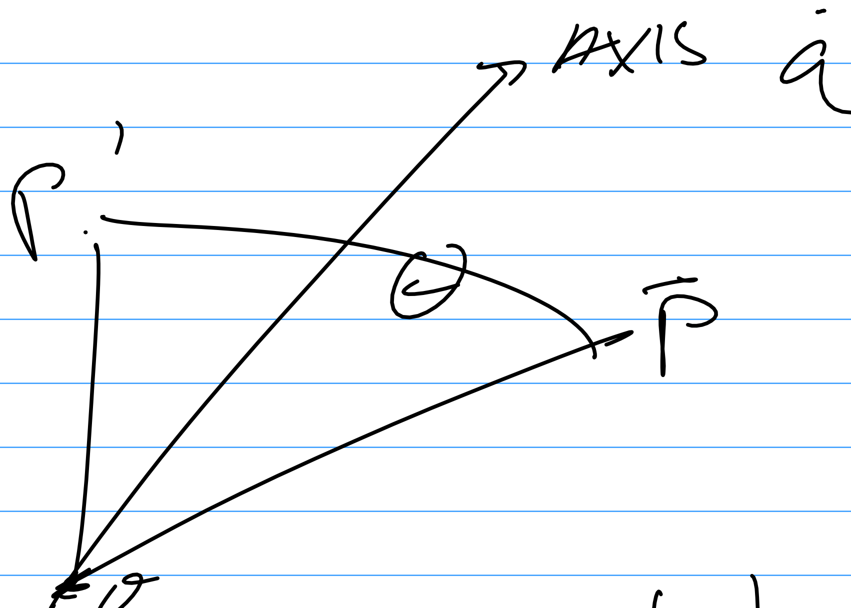


$\theta = 90^\circ \quad \cos = 0 \quad \sin = 1$

$$M = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

3D ROTATIONS



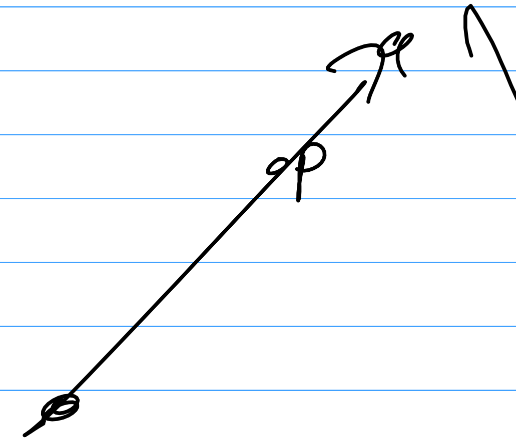
a IS NORMALIZED $|a|=1$

IF $a=(1,2,3)$ $|a|=\sqrt{14}$

$\rightarrow a=\left(\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}\right)$ $|a|=1$

TOO HARD. START WITH SIMPLE CASES

1. P IS ON AXIS.
   $p'=p$



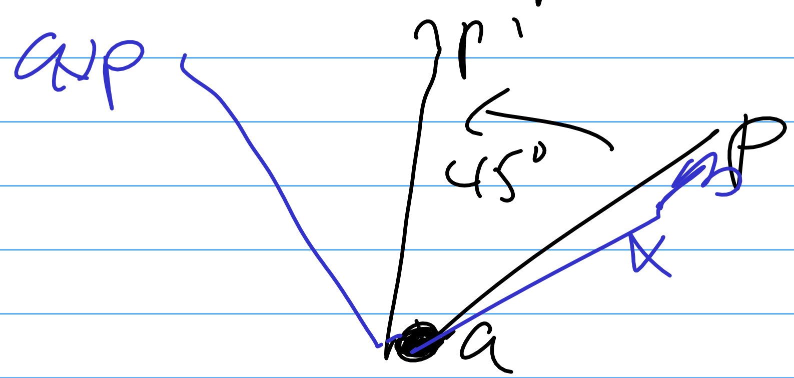
$$a = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

$$P = (100, 200, 300)$$

$$P' = P$$

EASY 2 $\quad P \perp a$

NOW THIS IS A 2D ROTATION IN PLANE $\perp a$



$P \perp a$

IN 2D SYSTEM, P IS LIKE AN X-AXIS

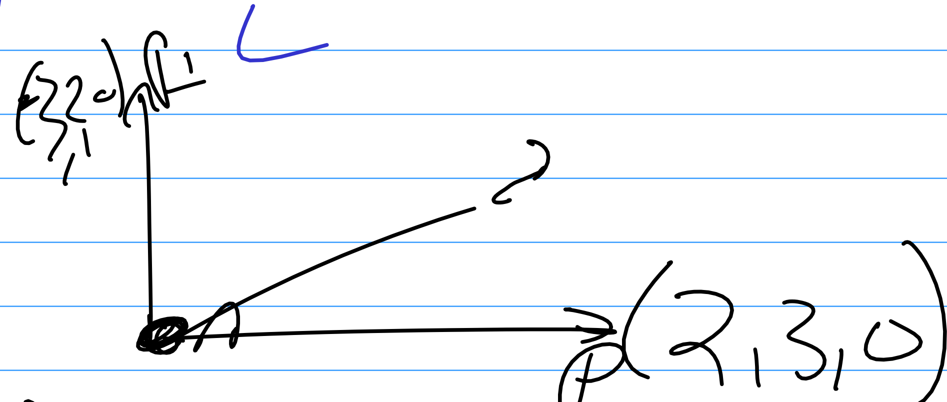
a×P IS LIKE A Y AXIS

$$P' = P\cos\theta + a \times P \sin\theta$$

WHEN I SAID THAT $p \perp a$ I MEANT THAT THE VECTOR FROM O TO P WAS $\perp a$

$$a = (0,0,1)$$
$$p = (2,3,0)$$
$$a \cdot p = 0 \qquad a \perp p$$
$$|a| = 1$$

Z PLAN



$$a \times p = \begin{pmatrix} 0, 0, 1 \\ 2, 3, 0 \end{pmatrix} \times (2,3,0) = (-3,2,0)$$

$$p' = (2,3,0)\cos\theta + (-3,2,0)\sin\theta$$

$\theta = 45^\circ \quad c = s = .7 \qquad p' = (-.7, 3.5, 0)$

SPLIT p INTO A PARALLEL COMPONENT + PERPENDICULAR COMPONENT

$$p_{\parallel} = a \cdot p \, a$$

$$p_{\perp} = p - a \cdot p \, a$$

DOT PRODUCT

$$a = (0,0,1)$$

$$p = (1,2,3)$$

$$a \cdot p = 3$$

$$p_{\parallel} = a \cdot p \, a = 3(0\,0\,1) = (0,0,3)$$

9

$$p_\perp = p - p_\parallel = (1,2,3) - (0,0,3)$$
$$= (1,2,0)$$

$$a \cdot p_\perp = (0,0,1) \cdot (1,2,0) = 0$$

$$p_\parallel' = p_\parallel$$

$$p_\perp' = p_\perp \cos\theta + a \times p_\perp \sin\theta$$

$$a \times p_\perp = a \times (p - a \cdot p\, a)$$
$$= a \times p - (a \cdot p)\, a \times a \quad (= 0)$$
$$= a \times p$$

$$p_\perp' = (p - a \cdot p\, a)\cos\theta + a \times p \sin\theta$$

$$p_\parallel' = p_\parallel = a \cdot p\, a$$

$$p' = p_{\parallel}' + p_{\perp}'$$

$$= a \cdot pa + (p - a \cdot pa)\cos\theta + a \times p \sin\theta$$

$$p' = p\cos\theta + a \cdot pa\,(1 - \cos\theta) + a \times p \sin\theta$$

EX/ $a = (1,0,0)$

$p = (3,2,1)$

$\theta = 45°$ $\cos = \sin = .7$

$a \cdot p = 3$ $a \times p$ 100
321

$= (0,-1,2)$

$$p' = .7(3,2,1) + .9(1,0,0) + .7(0,-1,2)$$

$$= (3, .7, 2.1)$$

PROBLEM OPENGL (AND OTHERS) WANT MATRIX

I WANT MATRIX M "

M IS COMPUTED FROM a, θ

$$p' = Mp$$

$$p' = p\cos\theta + \_\_\_\_ + \_\_\_\_$$

EASY MATRIX

$$M_1 = \begin{pmatrix} \cos\theta & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & \cos\theta \end{pmatrix}$$

$$p_1' = M_1 p$$

$$p' = \_\_\_\_ + a \cdot p\, a\, (1 - \cos\theta) + \_\_\_\_$$

I WANT A MATRIX M_{2a}

$$M_{2a} p = a \cdot p a$$

M_{2a} COMPUTED FROM ONLY a

$$M_{2a} = \begin{pmatrix} a_1^2 & a_1 a_2 & a_1 a_3 \\ a_1 a_2 & a_2^2 & a_2 a_3 \\ a_1 a_3 & a_2 a_3 & a_3^2 \end{pmatrix}$$

ES. $a = (1, 2, 3)$

$$M_{2a} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

$(1,2,3) \otimes (1,2,3) = M_{2a}\, p$

$p = (4, 5, 6)$

$a \cdot p = 32$

$a \cdot p\, a = 32\,(1,2,3) = (32, 64, 96)$

$$M_{2a}\, p = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix} \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 32 \\ 64 \\ 96 \end{pmatrix}$$

$$M_2 = M_{2a}(1-\cos\theta)$$

$$p' = \_\_\_ + \_\_\_ + (a \times p)\sin\theta$$

WANT $M_{3a} \Rightarrow M_{3a}\, p = a \times p$

$$M_{3E} = \begin{pmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -3 \end{pmatrix}$$

$a \times p$

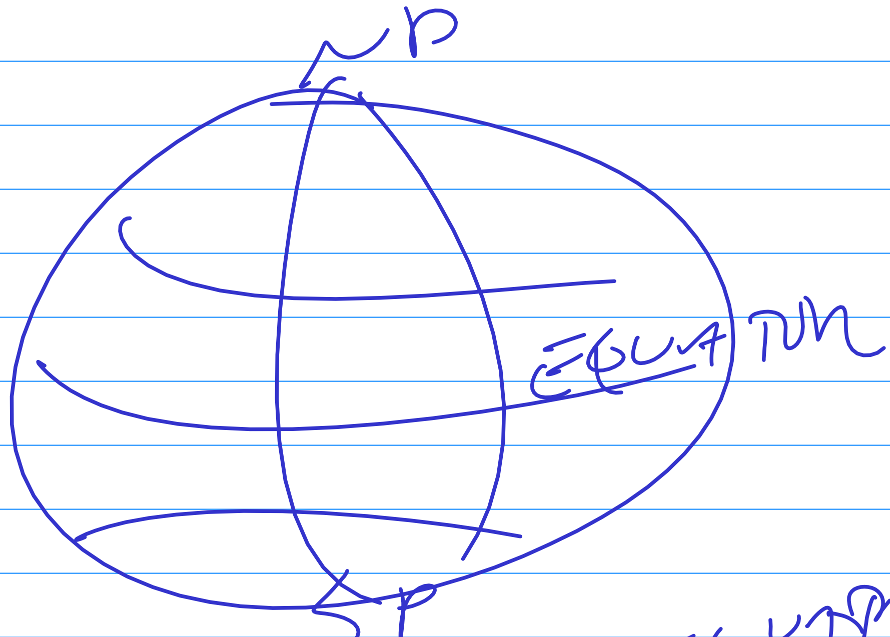
$$\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{matrix} \qquad a \times p = \begin{pmatrix} 3 \\ 6 \\ -3 \end{pmatrix}$$

PROBLEM WITH EULER ANGLES & WITH USING (LATITUDE, LONGITUDE)

B

ON EARTH.

IF YOUR PHYSICAL PATH IS MOVING SMOOTHLY THEN YOU WANT YOUR COORDINATES TO CHANGE SMOOTHLY



LOOK FROM POLARIS