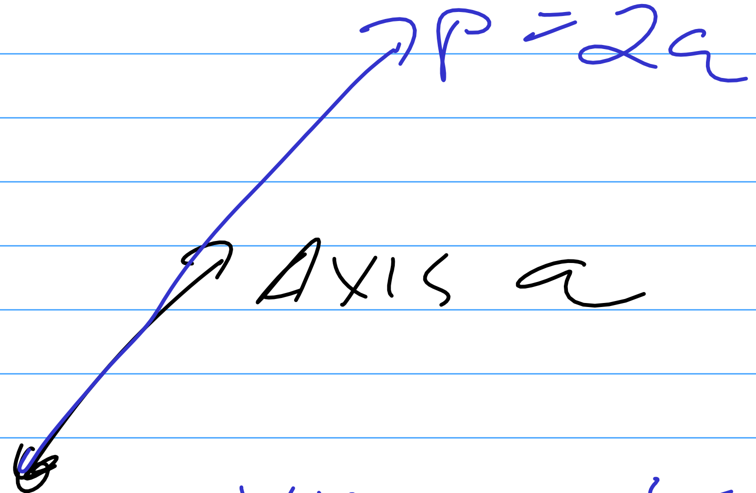
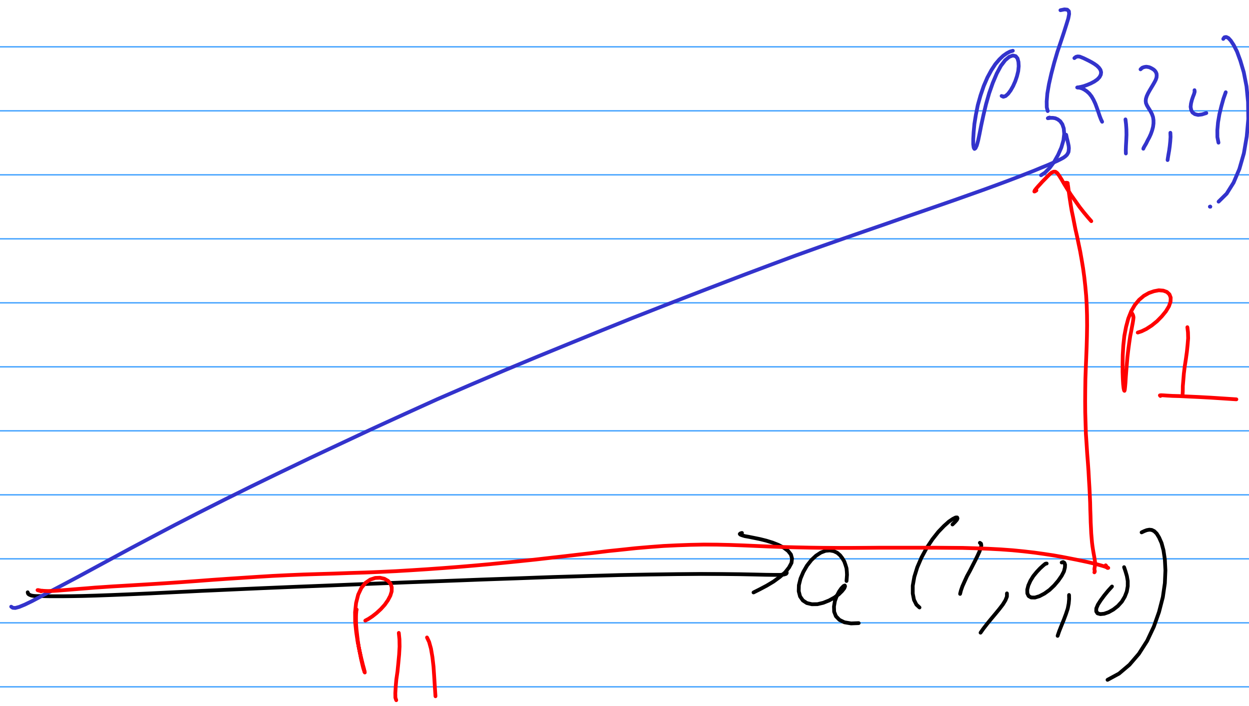
$p' = p$



POINTS ON AXIS DON'T MOVE WHEN YOU ROTATE THEM

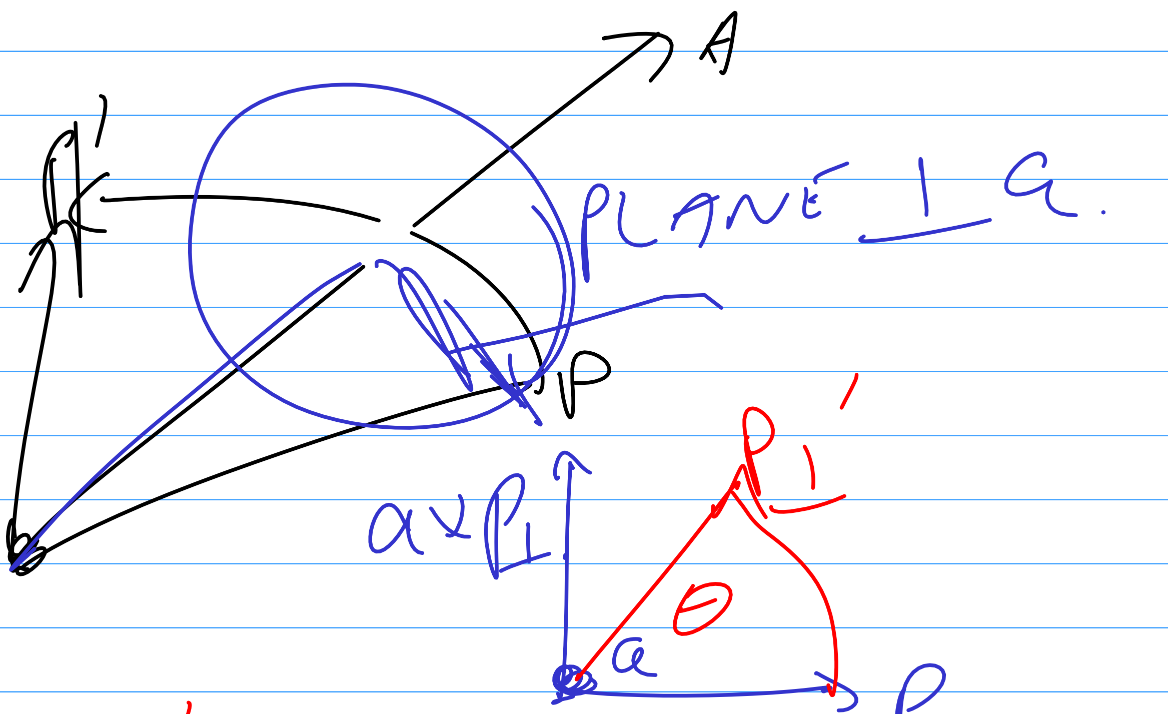


$$p = p_\parallel + p_\perp$$

$$p_\parallel = \underbrace{p \cdot a}_{2} \ \underbrace{a}_{(1,0,0)} = (2,0,0)$$

$$P_\perp = P - P_\parallel$$

$$= (2,3,4) - (2,0,0)$$



$$P_\perp' = \cos\theta \, P_\perp + \sin\theta \, a \times P_\perp$$

$$a \cdot p a = M_{3 \times 3} \, p$$

$$a \times p = M_{3 \times 3} \, p$$

GIVEN M

IS IT A ROTATION?

3x3 CART

PROPERTIES OF A ROTATION.

IT'S RIGID.

DISTANCES DON'T CHANGE

ANGLES

HANDEDNESS / PARITY / CHIRALITY DOESN'T CHANGE

HOW TO REALIZE?

ASIDE: A TRANSLATION IS ALSO RIGID. HOW DO I TELL THEM APART?

NOTE: A TRANSLATION MATRIX IS 4x4 HOM MATRIX:

$$\begin{pmatrix} 1 & 0 & 0 & D_x \\ 0 & 1 & 0 & D_y \\ 0 & 0 & 1 & D_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A HOM ROTATION IS THIS.

$$\begin{pmatrix} \sim & \sim & \sim & 0 \\ \sim & \sim & \sim & 0 \\ \sim & \sim & \sim & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

TRANS

GENERAL MAT

ROT+SCAL

$$\left(\begin{array}{c|c} 3\times 3 & 3\times 1 \\ \hline 1\times 3 & 1 \end{array}\right)$$

WEIGHT

PROJECTION

I'VE EXTRACTED THE 3x3 PART FROM THE 4x4 HOM TRANS MAT. IS THE 3x3 PART A ROTATION?

ROTATING $\begin{pmatrix}1\\0\\0\end{pmatrix}$ GIVES A VECTOR OF LEN = 1.

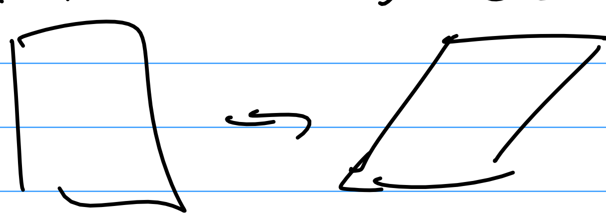
$$\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{32} \end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix} m_{11} \\ m_{21} \\ m_{31} \end{pmatrix} \triangleq C_1$$

IF M IS ROT $\Rightarrow$ $|C_1| = 1$

$|C_2| = |C_3| = 1$

$C_1 \cdot C_1 = 1$ ETC.

BUT M MIGHT STILL BE A SKEW



SO I HAVE TO CHECK ANGLES.

X AXIS $\perp$ Y AXIS BEFORE

AFTER $C_1 \cdot C_2 = 0$

$$C_1 \cdot C_3 = C_2 \cdot C_3 = 0$$

COMBINED $C_i \cdot C_j = \delta_{ij}$

$$\delta_{ij} \triangleq \begin{cases} 1 \text{ IF } i = j \\ 0 \text{ IF } i \neq j \end{cases}$$

DIRAC DELTA

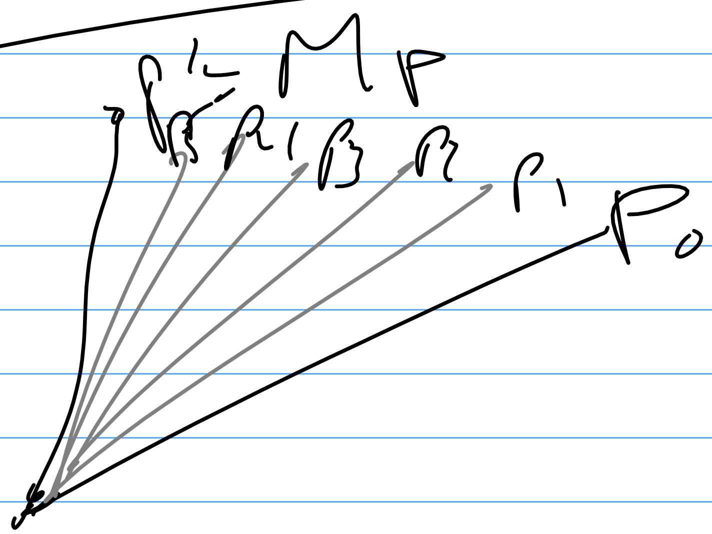
1 MORE THING - DID M REVERSE PARITY? $M = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

TEST $|M| = 1$

M IS ROTATION IFF :
- $C_i \cdot C_j = \delta_{ij}$
- $|M| = 1$

Q WHY DO YOU WANT AXIS + ANGLE?

A ANIMATION



WANT TO GO FROM P TO Mp IN 10 SMOOTH STEPS

HAVE TO FIND $M^{\frac{1}{10}}$

$p_1 = M^{\frac{1}{10}} p \qquad p_2 = M^{\frac{1}{10}} p_1$

- . . . -

IF YOU HAVE AXIS a
+ ANGLE θ
DO 10 STEPS WITH
OLD AXIS a + NEW
ANGLE $\frac{\theta}{10}$

HOW TO FIND AXIS + ANGLE OF M?
USES EIGENVALUES

THEY CAPTURE PHYSICAL PROPERTIES OF M
REGARDLESS OF COORDINATES.

USUALLY, M CHANGES A
VECTOR'S LENGTH AND DIRECTION.

2x2 EXAMPLE.

$$M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$Mv = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

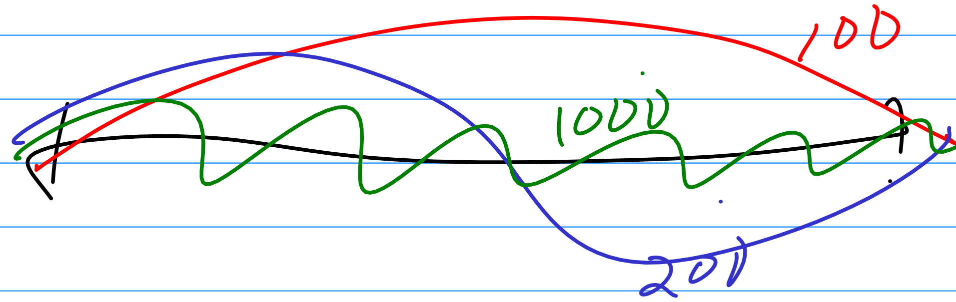
BUT IF $v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$Mv = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

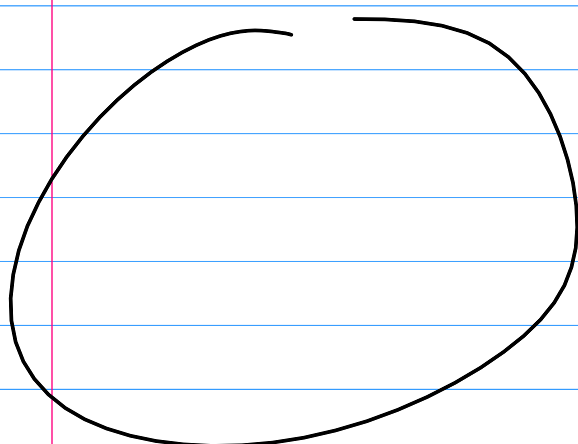
v DIDN'T CHANGE!

IF v DOESN'T CHANGE DIRECTION IT'S CALLED AN EIGENVECTOR OF M. HOW MUCH IT SCALES IS CALLED EIGENVALUE.

YOU CAN FIND THESE FOR A LOT OF PHYSICAL SYSTEMS.



VIOLIN STRING. FREQUENCIES ARE EIGENVALUES.



DRUM SKIN

EIGENVALUES ARE NOT INTEGER MULTIPLES

$N \times N$ MATRIX HAS N EIGENVALUES MAY BE DUPLICATES MAYBE COMPLEX CONJUGATE PAIRS.

HOW USEFUL FOR ROTATION?

$\mathbb{R}$

$$\underbrace{M v}_{\text{E.VECT}} = \underbrace{\lambda}_{\text{EVAL.}} v$$

SINCE ROTATIONS DON'T STRETCH OBJECTS, REAL EIGENVALUES = 1

$Mv = v$

POINTS ON AXIS ARE EIGENVECTORS WITH EIGENVALUE = 1.

TO GET ANGLE

OTHER 2 EIG. VALUES

$\cos\theta \pm i \sin\theta$

YOU CAN FIND AXIS + ANGLE USING EIGEN VECTORS + VALUES.

BUT IT'S EASIER

FOR ANGLE

$$m_{11} + m_{22} + m_{33} = 1 + 2\cos\theta$$

THERE ARE PROBLEMS WITH AMBIGUOUS ANGLES.

IF $\sin\theta = .7$

$\theta = 45°$ OR $135°$

IF $\cos\theta = -.7$

I WON'T WORRY ABOUT ANGLE AMBIGUITY BUT IF YOU WRITE A PROGRAM YOU'LL NEED TO

COMBINING ROTATIONS GIVES A NEW ROTATION WITH ITS AXIS + ANGLE.

$$M_3 M_2 M_1 = M$$