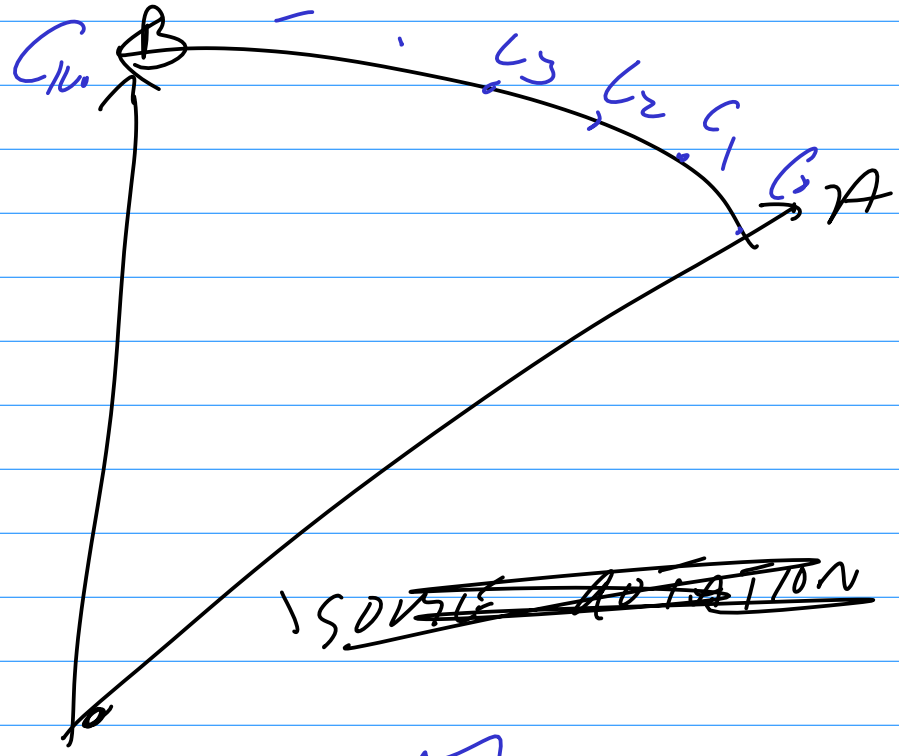
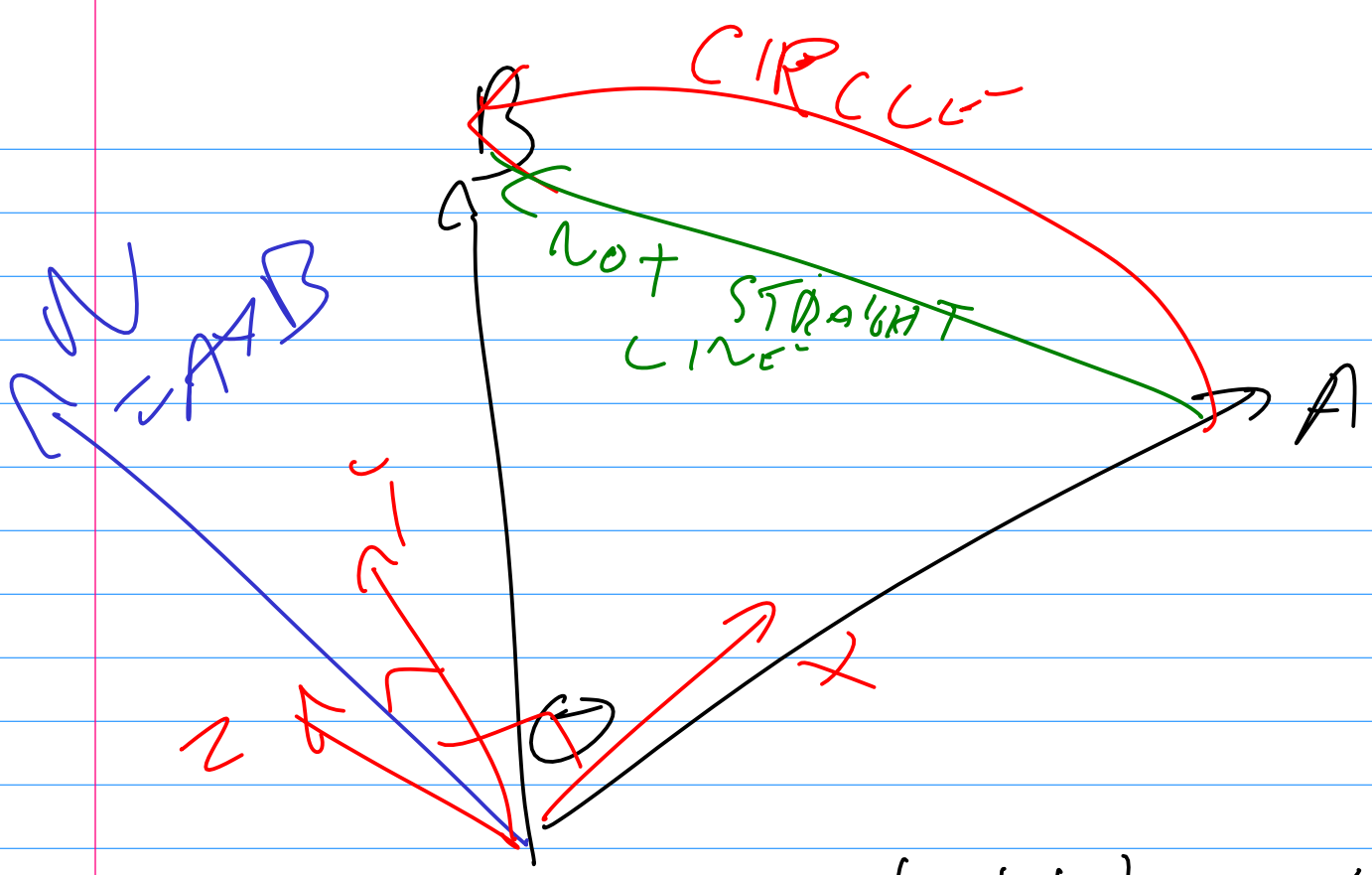



COMPUTE MATRIX M TO ROTATE
A → B NEED $M^{-1}$




FIND $\theta$: $A \cdot B = |A||B| \cos\theta$

IF $|A|=|B|=1$ $\cos\theta = A \cdot B$

WE HAVE A 3D COORDINATE SYSTEM WITH

$A$, $N \times A$, $N$

ROTATING AROUND $N$

$B =$

# ROTATIONS IN 2D

POINT $(x, y)$ $(3,5)$ $\longrightarrow$ COMPLEX N. $3+5i$

$z = x + iy$

TRANSLATE BY $(D_x, D_y)$ $(2,1)$ $+ (D_x + iD_y)$

$2+i$

$\rightarrow 5+6i$

ROTATE BY $\theta$ ABOUT ORIGIN $\rightarrow \times e^{i\theta}$

$\theta$ IN RADIANS

ROTATE BY $90^\circ = \frac{\pi}{2}$

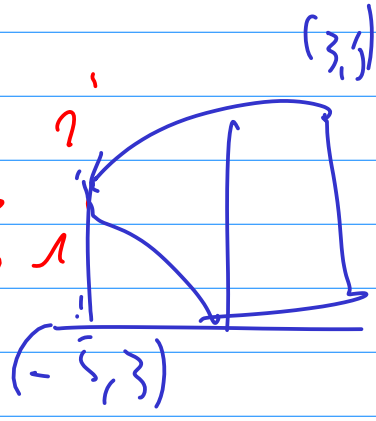
$\rightarrow$ MULT BY $e^{i\frac{\pi}{2}} = i$

TO ROTATE $(3,5)$ BY $90^\circ$,

MULTIPLY $3+5i$ BY $i$

$= 3i + 5i^2 = -5 + 3i$

POINT $(-5, 3)$



WE CAN EASILY COMBINE OPS I WANT TO

1. TRANS BY (1,2)
2. ROTATE BY 90°
3. TRANS BY (3,4)
4. SCALE BY 2
5. ROTATE BY 180° = π ($e^{i\pi} = -1$)

$$z' = 2(((z + (1+2i))i + (3+4i)) \cdots$$

$$iz + (-2+i)$$

$$iz + ((1+5i)$$

$$z' = -2iz + (-2-10i)$$

= 90° ROT
+ SCALE BY −2
+ TRANS BY (−2, −10)

Using complex numbers for 2D points lets you simplify a long string of translations, rotations, and scalings down to one scaling, one rotation, and then one translation.

Hamilton wanted a 3D analog to complex numbers.

Problem: 3D rotations don't commute.

---

REAL NUMBERS

$$Q = a + bi + cj + dk$$

INDETERMINATES

SIMPLIFY COMBOS OF $i, j, k$.

$$i^2 = j^2 = k^2 = -1$$

$$ij = k \quad jk = i \quad ki = j$$

$$ji = -k \quad kj = -i \quad ik = -j$$

$$Q_1 = 1 + 2i + 3j + 4k$$

$$Q_2 = 5 + 6i + 7j + 8k$$

$$Q_1 + Q_2 = 6 + 8i + 10j + 12k$$

$$7Q_1 = 7 + 14i + 21j + 28k$$

$Q_1 = 1 + 2i$

$Q_2 = 3j + 4k$

$Q_1 Q_2 = (1 + 2i)(3j + 4k)$

$= 3j + 4k + 6ij + 8ik$

$ij = k \qquad ik = -j$

$= 3j + 4k + 6k - 8j$

$= -5j + 10k$

POINT $(1,0,2)$

$p = 1 + 2k$

ROT BY $90^\circ$ ABOUT X AXIS

$Q = .7 + .7i$

$p' = (.7 + .7i)\, p\, (.7 - .7i)$

$= (.7i + 1.4k - .7 - 1.4j)\,(1+2k)\,(.7 - .7i)$

$= .5\,(i + 2k - 1 - 2j)(1 - i)$

$= .5\,(-1 + i - 2j + 2k)(1 - i)$

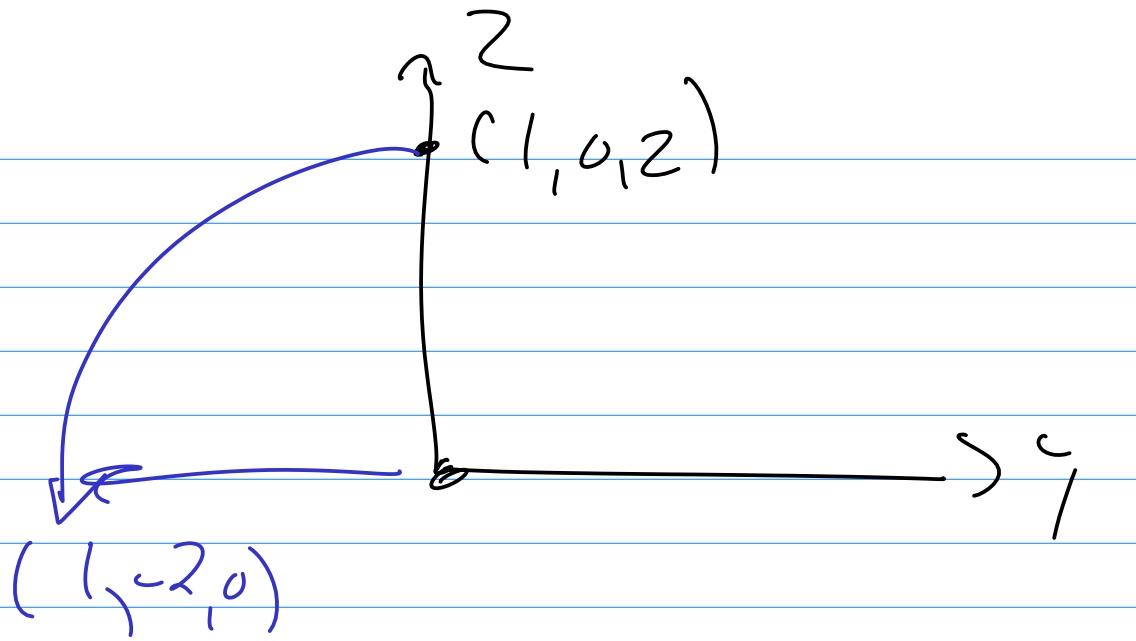
$= .5\,(-1 + i - 2j + 2k + i + 1 - 2k - 2j)$

$= .5\,(2i - 4j) = (i - 2j)$

POINT $(1, -2, 0)$

$(1,0,2) \longrightarrow (1,-2,0)$

$90^\circ$ ABOUT X AXIS




WHAT IS COMBO OF
90° ABOUT x THEN
90° ABOUT y ?

$$Q_1 = .7 + .7i = .7(1+i)$$

$$Q_2 = .7(1+j)$$

$$Q = Q_1 Q_2 = .7(1+i).7(1+j)$$

$$= .5(1+i+j+\cancel{ij}\ k)$$

$$= .5 + .5(i+j+k)$$

FINAL ROTATION

$$Q = .5 + .5(i + j + k)$$

$\cos\frac{\theta}{2} = \frac{1}{2}$

$\frac{\theta}{2} = 60^\circ$ $\quad \theta = 120^\circ$

$\frac{\sqrt{3}}{2}\left(\frac{i+j+k}{\sqrt{3}}\right)$

$120^\circ$ ABOUT $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$