The RPI GeoStar Project

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• DARPA-sponsored project at RPI
• March 2005 — Jan 2009
• continuing my ideas of many years
• Faculty: WRF, Prof Barb Cutler, Prof Frank Luk
• PhD students completed: Dr Metin Inanc, Dr Dan Tracy
• PhD students in progress: Chris Stuetzle, Zhongyi Xie
• MS students: Jake Stookey, Jon Muckell
• Contractors: Joe Roubal (ESRI)
Tasks

- Terrain representation
  - Morphological terrain sculpting
  - Overdetermined Laplacian PDE (ODETLAP)
  - Triangulated irregular network (TIN)
  - Lossy compression of terrain and slope

- Terrain operators
  - Siting/intervisibility toolkit
  - Trajectory (aka path, motion) planning
  - Drainage analysis
Slope Accuracy on Compressed Terrain — Why consider slope?

Slope is important for

- mobility
- erosion
- aircraft
- visibility
- recognition
Bad commercial slope representation

Commercial SW:
Bad commercial slope representation

Commercial SW:

Photo:
Inconsistent layer representation
Accurate elevations $\not\Rightarrow$ accurate slopes

- **Ignoring errors**, slope is simply $f'(x)$
- **But** $\limsup_{i \to \infty} \left| (f_i(x) - f(x)) \right| \to 0$, **gives no guarantees** about $\limsup_{i \to \infty} \left| (f'_i(x) - f'(x)) \right|$
- Consider two approximations to $y(x) = 0$
- Elevation got better but slope got worse.
ODETLAP – Overdetermined Laplacian Method

Fundamental representation for this work

- Small set of posts $\Rightarrow$ complete matrix of posts
- Overdetermined linear system:
  - $z_{ij} = h_{ij}$ for known points,
  - $4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}$ for all nonborder points.
  - Emphasize accuracy or smoothness by weighting the two types of equations differently.
- Original goal: fill contours to a grid w/o showing terraces; competing methods have these problems:
  - Information does not flow across contours $\Rightarrow$ slopes discontinuous
  - If rays are fired from the test point to the first known point, then method is not conformal etc.
ODETLAP Advantages

Handles

- missing–data holes.
- incomplete contours,
- complete contours,
- kidney–bean contours,
- isolated points,
- inconsistent data.
ODETLAP hard example

- input: contours with sharp corners
- output: smooth silhouette edges, inferred top
ODETLAP process

Input

400x400 matrix of elevations

ODETLAP point selection

contour lines

any user-supplied points, even inconsistent

Small point set ~1000

ODETLAP terrain reconstruction

Compressed distributed data

Reconstructed data

400x400 matrix of elevations
Goal is min size not fewest points

- What about not selecting points adaptively, but using a regular grid?
- More points, but all the \{ (x, y) \} combined compress down to one byte
- However, small features get lost, and
- that fact is not captured by a simple RMSE metric.
• *Coding* \{(x, y, z)\} to minimize size is as important as selecting the points.
• Using more points is good, if they can be coded better.
• E.g., regular grid of points.
• If progressive transmission is not desired, then, for irregular points, use compressed bitmap (CCITT G4) for \{(x, y)\} and *bzip2* for \{(z)\}. 
Information theoretic limit for point coding

• Assume that 1000 of the $400^2$ bits are 1, the others 0.
• Assume no further structure.
• Info content: $\log \left( \binom{160000}{1000} \right) = 8754$ bits.
• We are within 20% of that.
• That’s why we separate $\{(x, y)\}$ from $(z)$. 
ODETLAP summary

Original Surface
(320 KB)

Compressed Surface
(4071 Bytes)

Average Absolute Error = 2.451
Maximum Absolute Error = 25.822
ODETLAP fills in missing data holes
Slope definition, accuracy

- Zevenbergen-Thorne \(( (p_{i-1,j} - p_{i+1,j}) \times (p_{i,j-1} - p_{i,j+1}) )_z \)
- \(p_{ij}\) not used

Limits of slope accuracy

- 1m elevation resolution
- 30m post spacing
- slope precision: \(\arctan \left( \frac{1}{30} \right) \approx 3\% \approx 2^\circ\)

Info content

- Slope’s autocorrelation distance is smaller than elevation’s
- However, slope has less relative precision.
Level-II sample datasets

400 × 400 elevation matrices, *elevation range*

Hill1 505m  
Hill2 745m  
Hill3 500m  

Mtn1 1040m  
Mtn2 953m  
Mtn3 788m
Idea 1: Pin down the elevation at sets of close points

- When inserting a point into known set, also insert some adjacent points
- *Thesis*: that will force the slope to be accurate there.
- Not really.
- *Analogy* Lagrangian interpolation.

Keep trying.
Idea 2: Extend ODETLAP

- Explicitly incorporate slope
- New overdetermined linear system:
  - unknowns: $z_{ij}$
  - known:
    - some $h_{ij}$,
    - some $\Delta_x h_{ij} \triangleq h_{i-1,j} - h_{i+1,j}$,
    - some $\Delta_y h_{ij} \triangleq h_{i,j-1} - h_{i,j+1}$,
  - for all nonborder points:
    $$4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}$$
  - for known $h_{ij}$: $z_{ij} = h_{ij}$
  - for known $\Delta_x h_{ij}$ and $\Delta_y h_{ij}$:
    $$z_{i-1,j} - z_{i+1,j} = \Delta_x h_{ij}$$
    $$z_{i,j-1} - z_{i,j+1} = \Delta_y h_{ij}$$
Mtn2 experiments

Slope error vs compressed file size

mtn2, ODE TLAP with slope equations, lossy encoding of delta-z

Uncompressed binary file: 320KB
Incrementally Compressing LIDAR data

video: Dan8-10-08-1.m1v
Can’t see through ridges
Viewshed uncertainty

A small change in the interpolation algorithm between adjacent posts changes the visibility of 1/2 the viewshed.
Lossy compression evaluation using observer siting

Original, hi-res, terrain

Reduce horizontal or vertical resolution.

Lo-res Terrain

Compute multiple observer siting.

Best possible observer siting, given the data

Compute cumulative visibility.

Expensive cumulative visibility index

Transfer these observers to the hi-res terrain and compute their new cumulative visibility index.

Best observer set; computed on & evaluated on the lo-res terrain

Best estimate of the cheap cumulative visibility index

Compare.
Path planning

- Combines various projects we’ve worked on.
- *multiobserver siting* + *path planning* + *surface compression with ODETLAP*

- **unique feature of our path planning:** plans around complicated obstacles (viewsheds) while minimizing complex non-symmetric objective:

- not a *metric*: $d(a, b) \neq d(b, a)$

$$C = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \cdot \left(1 + \max\left(0, \frac{\Delta z}{\sqrt{\Delta x^2 + \Delta y^2}}\right)\right) \cdot (1+100v)$$
Lossy compression evaluation with path planning

Smugglers’ Path Planning on 16x Compressed “Scooped” Terrain Representation

Original 3595x3595 W111N31 Terrain: 12,924,025 d.f., Elev Range=2071

Compressed (7x7 Scoop): 791,267 d.f. (16x reduction), Mean abs error=1.7 (0.1%).

Compressed: Shortest Smugglers Path Computed Avoiding All 324 Viewsheds of Optimally Sited Observers

Original: Joint Viewshed Computed for Same 324 Observers

Evaluation: Optimal Path from Compressed Terrain Tested on Original Terrain Viewsheds – 14 of 4767 Points (0.3%) Are Erroneously Visible

WR Franklin, F Luk, C Westort, M Inanc, Z Xie, D Tracy

4/2006
Many optimal paths

video: RPI-multipath.wmv
Path planning on Ottawa LIDAR data
Path planning video

video: RPI-path-planning
Future Ideas

- conflate global lo-res with local hi-res elevation
- play red-blue games with multi-observer siting and path planning, detect and block choke points
- new apps for siting: radio transmitters, micro cells, exit lights, surveillance cameras
- urban multi-observer siting, in 3D
- conflation, compression and data fill-in of urban geometric data while preserving structure and the laws of formation
  - roads, rivers are continuous and usually don’t dead-end
  - size depends on catchment
- **Long term goal**: procedural terrain representation, where the math captures the structure.
- modify the real world to enhance the goal (visibility, motion, ...)

Commercialization

Two 2007.2 SBIRs clearly based on this research.

- **A07-123 Novel Representations of Elevation Data**
  Two phase I awards:
  - *W9132V-08-C-0012* to Andrews Space, Inc.
  - *W9132V-08-C-0013* to Numerica Corp.

- **A07-126 Optimal Intervisibility Site Selection**; cited me four times. Phase I award *W9132V-08-C-0005* to Toyon Research Corp.

(Unfortunately) I have no connection to any of those companies.
Followon projects

- Hosted the 18th Fall Workshop in Computational Geometry at RPI on Fri 10/31 and Sat 11/1/2008, with NSF support.
- Received an NSF Cyber-enabled Discovery and Innovation (CDI) Fundamental Terrain Representations and Operations award. (success rate: 2% (1 in 50)).
- Bathymetry: trackline fitting.
Sea floor bathymetry trackline surface fitting without visible artifacts using ODETLAP

*Problem:* Trackline data is very unevenly spaced, leading to very bad surface fitting.
More bad methods

Voronoi Polygons

Inverse Distance Weighting
Another bad, and ODETLAP

2nd-order Spline Inter
(w Matlab griddata)

ODETLAP, $R = 50$
Trackline data characteristics

- no wide-area data capture techniques such as IFSAR
- Multibeam Bathymetry produces many data points with 10m resolution in a swath up to 10km wide along a ship’s trackline.
- No data between tracklines.
- Test data: 10° × 10° region
- Want a 601 × 601 grid at 1’ cell resolution.
- 857 670 data points distributed among 23 630 grid cells.
- Average the average of 36 points per cell.
- 1/3 of cells had depth range > 100m.
- Therefore features smaller than that may not exist.
- Eval that with a combo of contours and relief shading using Mathematica.
Extending ODETLAP to variable $R$

*Problem:* The true surface may have varying detail levels, but probably not correlated with the tracklines.

*Soln:* Vary $R$ (the relative weights of the two equation types) according to the distance to the closest known points.

*Result:* Surface shows more details but few artifacts.

ODETLAP, Variable $R$ from 10 to 100